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Stability analysis, dynamical behavior and analytical solutions of nonlinear fractional differential system arising in chemical reaction

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ABSTRACT

In chemical reaction process, mathematical modeling of certain experiments lead to Brusselator system of equations. In this article, the dynamical behaviors of reaction Brusselator system with fractional Caputo derivative is studied. Also, Its stability and chaotic attractors of the commensurate fractional dynamical Brussleator system are discussed. The fractional derivative operators are nonlocal and having weak singularity as compare to the classical derivative operators. To find the analytical solutions of fractional dynamical systems is a big challenge, therefore, new techniques are worth demanding to solve such problems. To overcome this difficulty, the optimal homotopy asymptotic method is extended in this study to the system of fractional partial differential equations. A numerical example is presented as well to investigate the convergence, performance, and effectiveness of this method.

1. Introduction

In last few years, fractional calculus (fractional integral and the fractional derivative) has become the center of many studies because of various applications growing rapidly and played a vital role in engineering and science [1–5]. Chaos theory is a backbone to investigate the dynamical behavior of differential systems. Recently, researchers are getting great interest to study the chaotic behavior of fractional dynamic systems. It has been observed that fractional derivative possess memory and describe the dynamics of many real phenomena in the more sophisticated way as compare to its integer-order system. It has been found that many integer order systems persist the chaotic behavior when these systems become fractional dynamical systems [6–12].

In last 30 years, several fractional differential systems have been found which exhibit chaotic behavior, for example the fractional order Chua's, Lorenz, Chen, Arneodo, Bloch with delay, differential systems and other fractional order chaotic systems [13–20].

An integer order dynamical system with differential equations occurs in the study of biological and chemical reactions. The Brussels school [21–25] studied the behavior of a non-linear oscillations in the chemical reaction model

$$\begin{aligned} \eta &\rightarrow U, \\ \omega + U &\rightarrow V + P, \\ 2U + V &\rightarrow 3U, \\ U &\rightarrow Q, \end{aligned}$$

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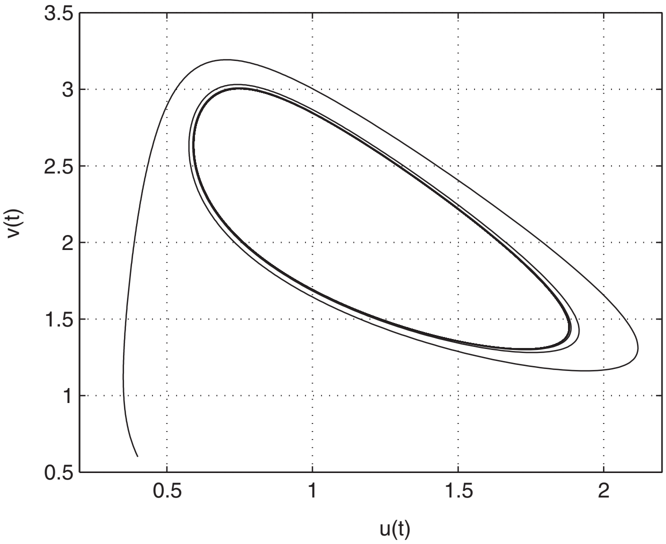

Fig. 1. The chaotic attractor of integer system (1).

where U and V are intermediates, η and ω are inputs, P and Q are outputs chemicals. Let $u(x, t)$ and $v(x, t)$ be the concentrations of U and V, and we assume that the concentrations of the input compounds η and ω are constant during the reaction process. This reaction Brusselator system plays a significant role in the study of cooperative processes of chemical kinetics. It has been observed that this reaction-diffusion Brusselator system arises in enzymatic reactions, in plasma and laser physics in numerous coupling among models, in the creation of ozone by atomic oxygen through a triple collision [26,27]. A pair of variables x and y are involved in dealing with chemical reactions, intermediates U and V with inputs η and ω and outputs P and Q chemicals, whose concentrations can be controlled during the reaction process under suitable conditions [22]. The integer order Brusselator differential system can be written as

$$\begin{cases} \dot{u}(x, y, t) = \eta + u^2 v - (\omega + 1)u, \\ \dot{v}(x, y, t) = -u^2 v + \omega u, \end{cases} \tag{1}$$

subject to the initial conditions

$$(u(x, y, 0), v(x, y, 0)) = (g_1(x, y), g_2(x, y)), \tag{2}$$

where $g_1(x, y)$, $g_2(x, y)$ are known functions, x, y and t denote spatial and temporal independent variables respectively, ω and η are constant concentrations of the two reactants.

When $\eta = 1$, $\omega = 2.25$, $(u(x, y, 0), v(x, y, 0)) = (0.4, 0.6)$, then Lyapunov exponents are $\lambda_1 = -0.0112$, $\lambda_2 = -0.3186$ and the chaotic behaviors are shown in Fig. 1. The dynamical behaviors of the integer order Brusselator system have also been studied in [28–30].

Taking the Caputo fractional derivative instead of integer order derivative, one can get the following time fractional system of differential equations, which known as the fractional Brusselator system in Caputo sense

$$\begin{cases} {}_C D_{0,t}^{\alpha} u(x, y, t) = \eta + u^2 v - (\omega + 1)u, \\ {}_C D_{0,t}^{\beta} v(x, y, t) = -u^2 v + \omega u, \end{cases} \tag{3}$$

where, $0 < \alpha, \beta < 1$, ${}_C D_{0,t}^{\alpha}$ and ${}_C D_{0,t}^{\beta}$ are fractional derivatives in Caputo sense. If $\alpha = \beta$, then system (3) is called commensurate fractional order Brusselator system, otherwise it is called the incommensurate.

In addition, to find the solution of fractional differential equations (FDEs) become hot topic these days. Several analytical and numerical methods have been proposed to find the solution of FDEs. Sarwar et al. [31–34] extended the optimal homotopy asymtotic method (OHAM) to solve the fractional order problems. Maryam et al. presented the 2D Brusselator system using method of lines [35]. Further, Singh et al. applied the q-homotopy analysis transform method (q-HATM) to solve the fractional Brusselator reaction-diffusion system which arising in triple collision and enzymatic reactions [26]. Bernstein polynomials has been used by Hasib et al. to find the numerical solution of fractional-order Brusselator [36]. Jafri et al. [37] applied variational iteration method (VIM) to fractional Brusselator System. Ongun et al. presented the nonstandard finite difference schemes to investigate the fractional order Brusselator system [38].

This paper is organised as: Section 2 is devoted for some basic definitions of fractional calculus. The stability analysis and dynamical behavior of nonlinear fractional Brusselator system is discussed in Section 3. To find the optimal analytical solution, the optimal homotopy asymptotic method is extended for system of nonlinear fractional dynamical system. The description of this extended method is elaborated in Section 4. To investigate the performance and effectiveness of this extended method, a numerical example with diffusion term is presented in Section 5. The numerical results obtained in Section 5 are discussed in Section 6. In the