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Scattering states solutions of Klein–Gordon equation with three physically solvable potential models

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ABSTRACT

The scattering state solutions of the Klein–Gordon equation with equal scalar and vector Varshni, Hellmann and Varshni–Shukla potentials for any arbitrary angular momentum quantum number l are investigated within the framework of the functional analytical method using a suitable approximation. The asymptotic wave functions, approximate scattering phase shifts, normalization constants and bound state energy equations were obtained. The non-relativistic limits of the scattering phase shifts and the bound states energy equations for the three potentials were also obtained. Our bound states energy equations are in excellent agreement with the available ones in the literature. Our numerical and graphical results indicate the dependence of phase shifts on the screening parameter β, the potential parameter b and angular momentum quantum number l.

1. Introduction

The solutions to relativistic equations are very essential in many aspects of modern physics, particularly, the Klein–Gordon equation is the most suitably used wave equation for the treatment of spinless particles in relativistic quantum mechanics. With various methods, Klein–Gordon equation has been studied with some exactly solvable potentials [1–11]. The scattering state solutions of various potential models have been a subject of considerable interest in the recent year because it gives an insight to understand atomic structures, electronic configuration of atoms, resonance and many collisions process [9–23]. However, the scattering state solutions of the Klein–Gordon equation with the Varshni, Hellmann and Varshni–Shukla potentials needed to be studied due to the roles of these potential models in many aspect of physics.

The Varshni potential is a short range repulsive potential energy function which has been used to describe the bound states of the interaction systems and has been applied in both classical and modern physics. Varshni potential is one of the potential energy functions that has been studied in the framework of the Schrödinger equation and it also plays a fundamental role in chemical and molecular Physics [24,25]. The Varshni potential function was studied by Lim using the 2-body Kaxiras–Pandey parameters. In his work, he reported that Kaxiras and Pandey used this potential to describe the 2- body energy portion of multi-body condensed matter [25].

The Hellmann potential [26–30] has been explored to explain the electron-core interaction [31], electron-ion [32], alkali hydride molecules and inner-shell ionization problem [33]. Very recently, Hamzavi et al. obtained the approximate solution of Hellmann potential within the framework of parametric Nikiforov–Uvarov method while the numerical result were confirmed using amplitude phase method [34]. Arda and Sever studied the approximate analytical solutions of the Dirac equation for Hellmann, Wei-Hua and Varshni potentials with their discussions centered on pseudospin and spin symmetric cases [35]. Onate et al. [36] looked into the

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approximate eigen solutions of DKP and Klein–Gordon equations with Hellmann via super-symmetric approach. The solution of Schrödinger equation with Hellmann was obtained by formula methods. The behaviour of the energy in the first three states were presented [36]. Also, in 2015, Yazarloo et al. extended the study to scattering states of Dirac equation for spin and pseudospin symmetries with Hellmann potential [37].

The Varshni–Shukla potential [38,39] has been used to obtain some spectroscopic constants [38] and also to evaluate some properties of Alkalis Chakogenide crystals in condensed matter physics [39]. Besides, Thakur and Mandal used this short range repulsive interaction to investigate the alkalis halide molecules [38]. Therefore, our aim is to study the scattering state solutions of Klein–Gordon equation with the equal scalar and vector Varshni, Hellmann and Varshni–Shukla potentials. The choice of equal scalar and vector potentials in this work is to ensure the solvability of the Klein–Gordon equation for independent boson reduces to the form of Schrödinger-like equation, where the spin of the particle is ignored due to exact cancellation [40]. The non-relativistic limit results are then obtained by following the condition imposed by Alhaidari et al. [41]. The scattering state solutions are discussed via functional analytical method.

This paper is organized as follows; Section 2 contains the basic equations including the Klein–Gordon equation with the equal scalar and vector, Varshni, Hellmann and Varshni–Shukla potentials. In Section 3, we investigate the scattering state solutions of the Klein–Gordon equation with the equal scalar and vector Varshni, Hellmann and Varshni–Shukla potentials. Section 4 contains the numerical and graphical results and finally, the conclusion is given in Section 5.

2. The basic equations

The Klein–Gordon equation with the scalar potential $S(r)$ and vector potential $V(r)$ in the natural unit $\hbar = c = 1$ is given [9] as

$$\{-\nabla^2 + [M + S(r)]^2\}\psi_{nlm}(r, \theta, \varphi) = [E_{n,l} - V(r)]^2\psi_{nlm}(r, \theta, \varphi) \quad (1)$$

where $E_{n,\ l}$ is the relativistic energy of the system and M is the rest mass of the spinless particles.

Using the Alhaidari et al. condition [41] for evaluating negative energy states from positive energy states and applying $\psi_{nlm}(r, \theta, \varphi) = r^{-1}U_{n,l}(r)Y_{lm}(\theta, \varphi)$ in Eq. (1), we obtain the time-independent radial Klein–Gordon equation for a spinless particles with the mixed scalar S(r) and vector V(r) potentials as:

$$\frac{d^2U_{n,l}(r)}{dr^2} + \left\{\left[\frac{1}{2}V(r) - E_{n,l}\right]^2 - \left[\frac{1}{2}S(r) - M\right]^2 - \frac{l(l+1)}{r^2}\right\}U_{n,l}(r) = 0. \quad (2)$$

The radial equation for spinless particles within equal scalar and vector potential, V(r) = S(r) may now be written as:

$$\frac{d^2U_{n,l}(r))}{dr^2} + \left[(E_{n,l}^2 - M^2) - (E_{n,l} + M)V(r) - \frac{l(l+1)}{r^2}\right]U_{n,l}(r) = 0 \quad (3)$$

Since the above radial equation has no exact solution, we apply an approximation

$$\frac{1}{r^2} \approx \frac{\beta^2}{(1 - e^{-\beta r})^2} \ or \ \frac{1}{r} \approx \frac{\beta}{(1 - e^{-\beta r})}, \quad (4)$$

throughout the study to deal with the centrifugal terms. This approximation has been reported to be valid for $\beta r \ll 1$ [42,43] and it resembles the Greene–Aldrich approximation used in Ref. [44].

3. Scattering state solutions

3.1. Scattering state solutions of Varshni potential (VP)

In this work, we consider the equal scalar and vector Varshni potentials [24, 25]

$$V_{VP}(r) = S_{VP}(r) = a\left[1 - \frac{b}{r}e^{-\beta r}\right], \quad (5)$$

where r is the internuclear distance, a and b are the strengths of the potential and β is the screening parameter which controls the shape of the potential energy curve. With approximation in Eq. (4), the Klein–Gordon equation with Varshni potential is transformed by the variable $z = 1 - e^{-\beta r}$, yielding

$$\frac{d^2U_{n,l}^{VP}(z)}{dz^2} - \frac{1}{(1-z)}\frac{dU_{n,l}^{VP}(z)}{dz} + \frac{1}{z^2(1-z)^2}[P_{VP}z^2 + Q_{VP}z + R_{VP}]U_{n,l}^{VP}(z) = 0, \quad (6)$$

with

$$P_{VP} = \frac{k_{VP}{}^2}{\beta^2} + l(l+1) - \frac{a(E_{n,l} + M)b}{\beta}, \ Q_{VP} = \frac{a(E_{n,l} + M)b}{\beta}, \ R_{VP} = -l(l+1), \quad (7)$$

where $k_{VP} = \sqrt{(E_{n,l}^2 - M^2) - a(E_{n,l} + M) - l(l+1)\beta^2}$ is the asymptotic wave number of the spinless particles in this Varshni potential. Assuming the wave function of the form