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Optical solitons, nonlinear self-adjointness and conservation laws for Kundu–Eckhaus equation

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ABSTRACT

In this article, Kundu–Eckhaus equation (KE) is studied from the perspective of modified tanh-coth method (MTC), extended Jacobi elliptic function expansion method (EJEF), Lie symmetry analysis, nonlinear self-adjointness and conservation laws (Cls). New soliton solutions like combined dark-bright, dark, periodic wave and singular soliton solutions are obtained. The equation is found to be a nonlinear self-adjoint, we construct the Cls using the new conservation theorem presented by Ibragimov. Physical interpretation for some of the obtained solutions are illustrated in Figures.

1. Introduction

In 1984 to 1985 Kundu [1] and Eckhaus [2,3] proposed the KE equation as a linearizable form of the nonlinear Schrödinger equation. Levi and Scimiterna [4] found out the relationship of the complex Burgers and the KE equations via Miura transformation, and the discretization of KE equation was found. Thereafter, a lot of researchers embark on investigating the soliton solutions of the KE equation for example, Wang et al. [6] have successfully applied Bäcklund transformation for obtaining bright and dark soliton solutions to the KE equation with the cubic-quintic nonlinearity. Clarkson, Tuszynski, Johnson, and Kodama have used the KE equation for defining some physical phenomena in the quantum field theory, weakly nonlinear dispersive water waves, and nonlinear optics [5,7,8]. Ganji et al. [9] have obtained some analytical solutions of nonlinear Radhakrishnan, Kundu and Laskshmanan equations using exp-function method. Moreover, exp-function method has been considered by some authors for obtaining analytical solutions of such physical models [10,11]. Taghizadeh et al. [12] have used the first integral method to the Eckhaus equation. Also, some authors have investigated the general structures of some important partial differential equations like the nonlinear Schrödinger equation which includes the KE equation as optical forms [13], water waves [14], fluid mechanics [15], viscous fluids [16], ion-acoustic waves [17], isotropic mediums [18], and optical fiber communications of physical phenomena [19]. Yang et al. use Ricatti–Bernouli sub-ODE to obtain some exact solutions for the KE equation [20]. More physical meaning, applications and interpretation of KE equations can be found in [21–47].

Furthermore, authors have been using different approach for the contruction of Conservation laws for different system of equation. These Cls have a lot of important staminas for the investigation of a physical system. One of these staminas for obtaining Cls is the explicit approach used by Laplace in 1798 [48]. Jacobi and Klein works gave a sitimulation to Emmy Noether to investigate about the possible relationship between Cls and symmetries of differential equations established out of variational principle. In 1915

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Noether discovered the existence of such a relationship and the theorem was published in 1918 [49]. However, this theorem by Noether is applicable to differential equations with a Lagrangian, so called Euler-Lagrange equations. For the sake of demonstration, Noether's theorem can not be used to the equation like evolution equations and to differential equations whose order is odd [49]. Moreover, these symmetries of Euler-Lagrange equations need to satisfy some additonal axioms [49,50].

In order to expunge from this limitations and problems, Ibragimov presented a new technique based on the formal Lagrangian concept and have the new conservation theorem proved. The definition of nonlinear self-adjointness of differential equations was introduced [50–52] and the formula for establishing local Cls was given. In summary, the Ibragimov theorem state that for any system of differential equations that satisfy the axioms of a nonlinearly self-adjointnees, it is permissible to use the formal Lagrangian for the estabishment of local Cls of a system corresponding to the systems symmetries. The KE equation is given by

$$i\Phi_t + \Phi_{xx} + 2(|\Phi|^2)_x\Phi + |\Phi|^4\Phi = 0. \tag{1}$$

In this work, our aim is to find a set of symmetries, nonlinear selfadjointness, Cls using new Cls technique and also to obtain new soliton solutions of KE equation using MTC [53] and EJEF [54] methods that are not obtained previously.

2. Description of the methods

2.1. MTC method

Consider the nonlinear PDE below

$$q_t = G(q, q_x, q_{xx}, \ldots) = 0. \tag{2}$$

Using $\xi = \mu(x - ct)$ and $q(\xi) = q(x, t)$ in Eq. (2), we get the following ODE:

$$-\mu c q'(\xi) = G(q(\xi), kq'(\xi), \mu q^2 q''(\xi)), \tag{3}$$

where $q(x, t)$ travels with speed c. The resulting ODE in Eq. (3) is solved by the MTC method, which admits the use of a finite series of functions of the form

$$q(x, t) = q(\xi) = a_0 + \sum_{J=1}^{N} [a_j Y^j(\xi) + b_j Y^{-j}(\xi)], \tag{4}$$

and the Ricatti equation

$$Y' = A + BY + FY^2. \tag{5}$$

Where A, B and F are constants and will be found later. The parameter N in Eq. (4) is a nonnegative constant that is found by balancing the linear term of highest order with the nonlinear term in Eq. (3). Putting Eq. (4) in the ODE in Eq. (3) and making use of Eq. (5), we have an algebraic equation in powers of Y. Because all of the coefficients of Y^jmust perish. This will give a system of algebraic equations with respect to parameters a_i, b_i, μ and c. With the use of Mathematica, we can determine a_i, b_i, μ and c. Eq. (5) has the special solutions [53] below:

- $A = B = 1, F = 0,\ Y(\xi) = e^{\xi} - 1$,
- $A = \frac{1}{2},\ F = -\frac{1}{2},\ B = 0,\ Y(\xi) = \coth(\xi) \pm csch(\xi)$ or $Y(\xi) = \tanh(\xi) \pm isech(\xi)$,
- $A = F = \pm\frac{1}{2},\ B = 0,\ Y(\xi) = \sec(\xi) \pm \tan(\xi)$ or $Y(\xi) = \csc(\xi) \pm \cot(\xi)$,
- $A = 1,\ F = -1,\ B = 0,\ Y(\xi) = \tanh(\xi)$ or $Y(\xi) = \tanh(\xi) \pm \coth(\xi)$,
- $A = F = \pm 1,\ B = 0,\ Y(\xi) = \tan(\xi)$,
- $A = 1,\ F = -4,\ B = 0,\ Y(\xi) = \frac{\tanh(\xi)}{1 + \tanh(\xi)^2}$,
- $A = 1,\ F = 4,\ B = 0,\ Y(\xi) = \frac{\tan(\xi)}{1 - \tan(\xi)^2}$,
- $A = -1,\ F = -4,\ B = 0,\ Y(\xi) = \frac{\cot(\xi)}{1 - \cot(\xi)^2}$,
- $A = 1,\ F = 2,\ B = \pm 2,\ Y(\xi) = \frac{\tan(\xi)}{1 \pm \tan(\xi)^2}$,
- $A = -1,\ F = -2,\ B = \pm 2,\ Y(\xi) = \frac{\cot(\xi)}{1 \pm \cot(\xi)^2}$.

Different values of Y can be derived for different arbitrary values of A, B and F.

2.2. Extended Jacobi elliptic function method

We present the main steps for the EJEF technique in this subsection, suppose that we have PDE as follows:

$$G(u, u_x, u_y, u_t, u_x y, \ldots) = 0. \tag{6}$$

We want to verify that the travelling waves are solutions of Eq. (6). We start by uniting the independent variables x, y and t to one particular variable by using the new variable