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Soliton dynamics for one dimensional quantum system incorporating higher-order dispersion effect and nonlinear interactions

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ABSTRACT

We investigate (1 + 1) dimensional higher order nonlinear Schrödinger equation with higher order nonlinear interaction which models systems incorporating higher order dispersion and nonlinear effects. Through the F-expansion method, we analytically solved the higher order nonlinear Schrödinger equation model andidentify bright soliton type solution under certain parametric setting, demonstrating the practical significance of the theoretical treatment presented in this study.

1. Introduction

Nonlinearity related physics study is an intriguing topic in current physical world investigation. Solitons, which arise from balancing effects between dispersion and nonlinear interaction are the typical nonlinear phenomena and nonlinear Schrödinger equation (NLSE) [1–4] has been proven to effective in describing many nonlinear phenomena of quantum nature successfully. For example, NLSE can be utilized to model wave propagation in fields like nonlinear optics [5], condensed-matter physics [6,7] and plasma physics [8].

While it is well known that NLSE incorporating leading order nonlinear interaction is pretty thoroughly investigated, in certain application scenarios like quantum optics and Bose–Einstein condensates (BEC) system [9–11], higher order nonlinear effects arising from higher power of refractive index expansion or three-body interaction should be incorporated in the theoretical study. Also, when higher order nonlinear effects participate in the system's evolutionary dynamics, the spatial-temporal scale of the associated nonlinear structure becomes so small that we should take higher-order dispersion effects into consideration, and to model this kind of situation, the fourth-order nonlinear Schrödinger equation is the ideal choice [12–16]. We anticipate that typical nonlinear feature like soliton should also appear arising from such higher order model.

In this paper,we will analytically study the (1 + 1) dimensional fourth order NLSE with higher-order interactions which contains two-body, three-body and higher-order wave scattering effects in addition to the second, third and fourth-order dispersive effects. Through *F*-expansion method [17,18], we analytically solve the higher order NLSE, and identify the bright soliton type solution under certain parametric setting, demonstrating the particular nonlinear feature that is supported by the higher order NLSE under investigation.

This paper is organized as follows. Section II describes the fourth order NLSE with higher-order interactions together with the *F*-expansion method for solving the equation model. Section III gives the procedure details of reaching analytical solution of higher order NLSE. In the last section, we give conclusive remarks.

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2. Fourth order NLSE model with higher-order interactions and modified *F*-expansion method

2.1. Fourth order NLSE model with higher-order interactions

The (1 + 1) dimensional Fourth order nonlinear Schrödinger equation with higher-order interactions takes the following form

$$i\frac{\partial\psi(x,t)}{\partial t}+\gamma_1\frac{\partial^4\psi(x,t)}{\partial x^4}+i\gamma_2\frac{\partial^3\psi(x,t)}{\partial x^3}+\gamma_3\frac{\partial^2\psi(x,t)}{\partial x^2}+g_0|\psi(x,t)|^2\psi(x,t)$$
$$+g_1|\psi(x,t)|^4\psi(x,t)+g_2\frac{\partial^2|\psi(x,t)|^2}{\partial x^2}\psi(x,t)=0 \tag{1}$$

where γ_1, γ_2 and γ_3 parameterize the dispersion terms of various order, with $\gamma_1 \neq 0$. g_0 and g_1 are the two-body and three-body nonlinear interaction strength coefficients respectively, and the term with the interaction strength coefficient g_2 is the contribution from higher-order scattering effects. Here the parameter $g_2=\frac{a^2}{3}-\frac{ar_e}{2}$, in which m is the atomic mass, r_e is the effective range, and a is s-wave scattering length [19].

2.2. The *F*-expansion method

We can utilize the *F*-expansion method to solve nonlinear partial differential equation of the general form like

$$G(u,u_t,u_x,u_{xx},\ldots)=0 \tag{2}$$

where $u(x, t)$ is the unknown function to be solved, and the G is the polynomial of $u(x, t)$ and its partial derivatives of various orders. The basic idea of the *F*-expansion method is using the polynomial of $F(\xi)$ to express $u(x, t)$ with $F(\xi)$ defined as

$$\left(\frac{dF(\xi)}{d\xi}\right)^2=F^4(\xi)+b_3F^3(\xi)+b_2F^2(\xi)+b_1F(\xi)+b_0 \tag{3}$$

where

$$\xi=px+qt \tag{4}$$

Here b_3, b_2, b_1, b_0, p, q are parametric constants.So we can assume that the $u(x, t)$ takes the following form

$$u(x,t)=\sum_{i=0}^{m}h_i(t)F^i(\xi) \tag{5}$$

By balancing between the highest derivative and nonlinear term in Eq. (2), we can determine the integer value m. Substituting Eq. (5) into Eq. (2) and utilizing Eq. (3), by setting the coefficient formula of all terms of the resultant polynomial (of F or its derivatives) to zero we obtain a group of ordinary differential equations (ODEs) for $h_i(t)$. By solving the ODEs consistently, we reach the explicit analytical solution expressed as (5). we will construct the analytical solution of Eq. (1) based on this approach.

3. Analytical solutions of the fourth order NLSE with higher-order interactions

We can solve Eq. (1) by assuming that $\psi(x, t)$ takes the following format:

$$\psi(x,t)=\varphi(\xi)\exp[i(At+Bx)], \tag{6}$$

where A and B are constants to be determined later. Substituting (6) into Eq. (1), we get

$$\psi_x=\varphi'p\exp[i(At+Bx)]+i\varphi\exp[i(At+Bx)]B \tag{7a}$$

$$\psi_{xx}=\varphi''p^2\exp[i(At+Bx)]+2i\varphi'p\exp[i(At+Bx)]B+\varphi\exp[i(At+Bx)](-B^2) \tag{7b}$$

$$\psi_{xxx}=\varphi'''p^3\exp[i(At+Bx)]+3i\varphi''p^2\exp[i(At+Bx)]B-3\varphi'p\exp[i(At+Bx)]B^2-i\varphi\exp[i(At+Bx)]B^3 \tag{7c}$$

$$\psi_{xxxx}=\varphi''''p^4\exp[i(At+Bx)]+4i\varphi'''p^3\exp[i(At+Bx)]B-6\varphi''p^2\exp[i(At+Bx)]B^2-4i\varphi'p\exp[i(At+Bx)]B^3$$
$$+\varphi\exp[i(At+Bx)]B^4 \tag{7d}$$

and Eq. (1) is updated as follows

$$iq\varphi'-A\varphi+\gamma_1(\varphi''''p^4+4Bi\varphi'''p^3-6\varphi''p^2B^2-4i\varphi'pB^3+\varphi B^4)$$
$$+i\gamma_2(\varphi'''p^3+3i\varphi''p^2B-3\varphi'pB^2-i\varphi B^3)+\gamma_3[\varphi''p^2+2i\varphi'pB+\varphi(-B^2)]$$
$$+g_0\varphi^3+g_1\varphi^5+2g_2p^2(\varphi'^2+\varphi\varphi'')\varphi=0 \tag{8}$$

The imaginary Part of Eq. (8) is

$$\varphi'(q-4\gamma_1pB^3-3\gamma_2pB^2+2\gamma_3pB)+\varphi'''(4\gamma_1p^3B+\gamma_2p^3)=0 \tag{9}$$