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# Analysis of multi-band pyrometry for emissivity and temperature measurements of gray surfaces at ambient temperature

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## HIGHLIGHTS

- An optimization algorithm is proposed for overdetermined multi-band pyrometry systems.
- Multi-band pyrometry results are improved over dual-band pyrometry.
- Correct band selection is essential for the improvement of multi-band pyrometry.
- Background temperature has a strong impact on multi-band pyrometry results.

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## ABSTRACT

A multi-band pyrometry model is developed to evaluate the potential of measuring temperature and emissivity of assumably gray target surfaces at 300 K. Twelve wavelength bands between 2 and 60 μm are selected to define the spectral characteristics of the pyrometers. The pyrometers are surrounded by an enclosure with known background temperature. Multi-band pyrometry modeling results in an overdetermined system of equations, in which the solution for temperature and emissivity is obtained through an optimization procedure that minimizes the sum of the squared residuals of each system equation. The Monte Carlo technique is applied to estimate the uncertainties of temperature and emissivity, resulting from the propagation of the uncertainties of the pyrometers. Maximum reduction in temperature uncertainty is obtained from dual-band to tri-band systems, a small reduction is obtained from tri-band to quad-band, with a negligible reduction above quad-band systems (a reduction between 6.5% and 12.9% is obtained from dual-band to quad-band systems). However, increasing the number of bands does not always reduce uncertainty, and uncertainty reduction depends on the specific band arrangement, indicating the importance of choosing the most appropriate multi-band spectral arrangement if uncertainty is to be reduced. A reduction in emissivity uncertainty is achieved when the number of spectral bands is increased (a reduction between 6.3% and 12.1% is obtained from dual-band to penta-band systems). Besides, emissivity uncertainty increases for pyrometers with high wavelength spectral arrangements. Temperature and emissivity uncertainties are strongly dependent on the difference between target and background temperatures: uncertainties are low when the background temperature is far from the target temperature, tending to very high values as the background temperature approaches the target temperature.

## 1. Introduction

The thermal radiation power emitted by a blackbody is distributed along the spectrum according to Plank's law [1,2]:

$$L_\lambda(T) = \frac{c_1}{\lambda^5(\exp\frac{c_2}{\lambda T} - 1)}, \qquad (1)$$

where $T$ is the temperature (K), $\lambda$ is the wavelength (μm), $c_1 = 3.7418 \times 10^8\ \mathrm{W\ \mu m^4\ m^{-2}}$, and $c_2 = 1.4388 \times 10^4\ \mathrm{\mu m\ K}$. A pyrometer absorbs thermal radiation, typically in the infrared (IR) range, from an emitting target surface onto a detector; its output signal is related to the temperature of the target surface according to a transfer function, normally obtained by the calibration with a blackbody. For a given temperature, as all real surfaces emit, proportionally to its emissivity ($\varepsilon$), less radiation than a blackbody, the pyrometer temperature output will be different from that of the target surface. Consequently, the value of $\varepsilon$ must be known, so as

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to enable one to infer the true temperature of the target surface from the pyrometer output.

Basically, multi-band pyrometry consists of a set of $n > 1$ pyrometers, whose detectors absorb radiation within different wavelength bands, simultaneously measuring the temperature of a target surface of unknown emissivity. If $n = 2$, these systems are normally referred to as dual-band pyrometers. The basic principle of any multi-band pyrometer is that the emissivity is modeled as a function with $m < n$ unknown parameters, so that a system of $n$ equations with $m + 1$ unknowns (surface temperature is the remaining unknown) can established. If $n = m + 1$, the system is determined, having a unique solution for temperature and emissivity function parameters [3]; if $n > m + 1$, the system is overdetermined, so that temperature and emissivity solutions have to be computed through some least squares technique [4]. Multi-wavelength models have been proposed when the spectral bands are very narrow, in which all incoming radiation is assumed to be absorbed at discrete wavelengths, avoiding the need to integrate Plank's distribution.

The simplest multi-band model is based on the ideal dual-wavelength ratio thermometer, in which $n = 2$ discrete wavelengths absorb radiation from an assumably gray target surface, and if reflected background radiation is ignored (background is assumed to be much colder than the target surface), by applying Wien's distribution law (approximation of Planck's distribution for short wavelengths), it results in a relatively simple analytical solution for temperature and emissivity [1,2,5,6]. The model suggests that measurement uncertainties become less significant with increasing band separation, although the gray surface assumption becomes less likely [3].

Higher order ($n > 2$) multi-wavelength pyrometry systems were developed to extend dual-wavelength systems to non-gray target surfaces. Simpler models employ emissivity functions with exactly $m = n - 1$ parameters (determined systems) [3,5,7–9]. However, Coates [8] reported that multi-wavelength determined systems with $n > 3$, even in the absence of measurement errors, lead to extremely large temperature uncertainties due to the effect of overfitting. Besides, Khan et al. [3] recommended the use of dual-wavelength over higher order multi-wavelength systems.

Coates [8] also suggested that overfitting can be overcome by increasing the number of wavelengths, so that $m < n - 1$, resulting in an overdetermined system of equations that could be solved by least squares. Various overdetermined multi-wavelength systems have been reported with fairly positive results [4,10–15].

Multi-wavelength systems assume that the detectors are sensitive to discrete wavelengths, albeit real detectors are sensitive to wavelength bands, requiring the integration of Plank's distribution over the wavelength bands of the detectors. Besides, many models ignore the effect of reflected background radiation [4,10–15], which is only valid if the background is much colder than the target surface [16,17]. As a result, more advanced dual-band and multi-band pyrometry systems have been proposed [17–27]. The multi-band model reported by Fu et al. [18] indicates that multi-band pyrometry systems may be preferred over multi-wavelength systems in the determination of surface temperature.

Araújo et al. [26] investigated and quantified how the emissivity output from dual-band pyrometry systems are affected by the spectral characteristics of the detectors and background temperature, concluding that emissivity uncertainty was minimized for narrow spectral bands, far apart from each other, located towards lower wavelengths, and when the difference between background and target temperatures increased. Araújo and Martins [27] undertook an identical investigation for the temperature output from dual-band systems and reported similar conclusions regarding the variation of temperature uncertainty. It is the purpose of the present paper to perform a similar analysis for higher order multi-band systems, i.e. to quantify the impact of the spectral characteristics and background temperature on both temperature and emissivity uncertainties. It is also intended to investigate how the number of spectral bands and their spectral arrangements affect temperature and emissivity uncertainties, particularly by comparing dual-band with higher order multi-band results.

## 2. Pyrometer characteristics

### 2.1. Spectral bands

The same principles applied by Araújo et al. [26] and Araújo and Martins [27] were used in the selection of the spectral characteristics of the IR detectors: the spectral bands of the detectors are within the spectral range where most radiation is emitted by the target surface, and all spectral bands receive an identical amount of radiation from the target surface, so as to guarantee identical signal-to-noise ratios [28]. Moreover, a unique target temperature $T_s = 300$ K was considered, since this is a typical ambient temperature.

Since 98.0% of the radiation power emitted by a blackbody at 300 K is between 2 and 60 μm, this wavelength range was divided into 13 wavelength regions of equal power (7.69% of the total power between 2 and 60 μm), and the wavelengths delimiting these regions were rounded to one decimal place, resulting in the following values (Fig. 1): 2.0, 6.8, 8.2, 9.3, 10.4, 11.6, 12.8, 14.2, 15.9, 18.0, 20.7, 24.7, 32.0, and 60.0 μm.

The spectral bands of the detectors were defined by considering all possible combinations of every two consecutive spectral regions from the 13 regions specified above, resulting in the 12 bands presented in Table 1. An index ($b$) between 1 and 12 was assign to each spectral band ($\lambda'$-$\lambda''$), representing its relative wavelength position in the spectrum. Table 1 also shows the radiation power fraction of each band ($x_b$) relatively to the radiation power emitted by a blackbody at 300 K between 2 and 60 μm. Only spectral bands of equivalent power (providing equivalent signal-to-noise ratios) were considered, since published results for dual-band pyrometry indicated that there is no advantage in using wider spectral bands [26,27].

### 2.2. Uncertainty

The uncertainty of commercial pyrometers is typically specified as $T \pm \Delta T$, where $T$ is the pyrometer temperature output, and $\Delta T$ is the maximum temperature by which the true temperature ($T_i$) can deviate from $T$. As recommended for Type B uncertainty evaluation, and since there is no specific knowledge about the probability of $T_i$ within interval $T \pm \Delta T$, it can only be assumed that $T_i$ is equally

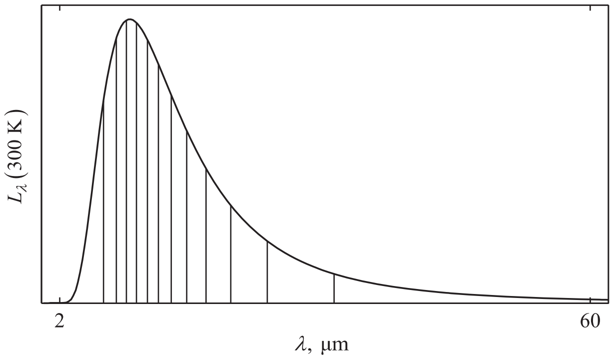

**Fig. 1.** Division of the spectrum between 2 and 60 μm into 13 wavelength regions of identical emitting radiation power from a 300 K blackbody.