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The splitting features of a frequency spectrum of a gyroscope based on elastic waves in solids: An isolated imperfect ring as an example

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Abstract

The calculation results on dynamic characteristics of a geometrically imperfect ring turning out of shape in its plane have been exemplified by a simpler computational model for a ring resonator of a gyroscope based on elastic waves in solids. The specific malconformations were shown to be responsible for a splitting of the bending frequency spectrum of such rings. In so doing the spectral mismatch may appear in cases different from the ideas of modern theory.

The splitting of the bending frequency spectrum was established to occur not only in the cases when the number of formative waves being equal to that of malconformation waves of the ring (as it is commonly believed at present) but also in the cases when the number of formative waves being two, three, four and so on times more than that of malconformation waves.

Keywords: Ring; Resonator; Wave solid-state gyroscope; Bending frequency spectrum; Radial oscillation.

Introduction

Creating new navigation and motion control systems and perfecting the existing ones is currently one of the key directions in the development of space technology, in particular, the GPS and GLONASS global satellite navigation systems [1–4]. Solid-state gyroscopes (SSG), used to construct inertial devices, show promise for their practical applications. The sensitive element of an SSG is a thin elastic axisymmetric ring resonator that makes bending normal-mode oscillations [4–7]. The error of these devices is due to the imperfections in their industrial manufacturing technology [8–15]. The disadvantages of these devices include the splitting of bending vibration frequencies, undesirable features in the vibrations of the rings [12–14], beating and drift in the wave pattern of the dynamic deformations of the resonator [15].

Some studies have been dedicated to the dynamic behavior of thin rings. However, the results of these studies to date do not always agree with the experimental data.

For example, it was established in Refs. [16–19] that the splitting of the bending frequency spectrum of shells and rings only occurs when the number of circumferential dynamic strain waves is equal to the number of shell (or ring) malconformation waves that

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Table 1
Comparison of the calculated data for the frequencies obtained by the finite-element method with the results of the analytical solution.

Number of circumferential waves n	Natural vibration frequency of an ideal ring f_n (Hz)					
	Calculation with MSC Nastran					Analytical solution [13,17]
	0.05[a] 126	0.03 209	0.02 314	0.01 628	0.005 1257	
2	3.123	3.122	3.121	3.121	3.121	3.871
3	8.832	8.829	8.829	8.828	8.828	9.623
4	16.935	16.93	16.928	16.927	16.927	17.31
5	27.387	27.378	27.375	27.373	27.373	27.72
6	40.177	40.162	40.158	40.154	40.154	40.64
7	55.292	55.275	55.27	55.265	55.265	55.81
8	72.738	72.716	72.708	72.702	72.702	73.42

[a] The top number refers to the size of the FE, the bottom number refers to the amount of FEs.

either coincide with the shape of the expected dynamic deflection or repeat it.

However, this conclusion is disputed in Refs. [20,21], considering, as an example, a simpler (limiting) problem of the oscillations of an infinitely long circular cylindrical shell that is a ring under plane strain. These studies have revealed that the splitting of the bending frequency spectrum occurs not only when the number of circumferential dynamic strain waves is equal to the number of ring malconformation waves (as it has been generally assumed at the present time), but also when the number of shape-generating waves is half the number of malconformation waves.

The present work is dedicated to numerical study of splittable bands of the bending frequency spectrum of a geometrically imperfect isolated ring. The results and conclusions of Refs. [20,21] were refined and complemented with new data.

Dynamic characteristics of imperfect rings

The MSC Nastran software package was used to simulate and solve the problem of oscillations of an isolated ring with the following parameters:

width $a = 0.005\,\text{m}$;
thickness $h = 0.005\,\text{m}$;
radius $R = 1\,\text{m}$ ($R/h = 200$);
Young's modulus $E = 2 \times 10^{11}\,\text{Pa}$;
mass density $\rho = 7800\,\text{kg/m}^3$;
Poisson's ratio $\mu = 0.3$.

One-dimensional finite elements (FEs) experiencing tension, compression, torsion, transverse shear and bending were used for modeling the ring. The number of FEs was chosen to ensure high accuracy of the calculation. This accuracy corresponds to a grid with 1257 finite elements (Table 1).

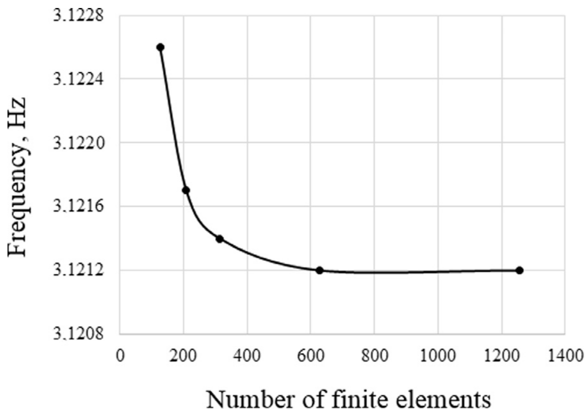

Fig. 1. Estimation of the convergence of the calculations of the natural vibration frequency of an ideal ring by the finite element method for the value $n = 2$ (the fundamental frequency of the spectrum, see Table 1).

The accuracy of the analytical solutions [13,17] is bound by the same limits as the accuracy of the initial equations of shallow shell theory, which lead to satisfactory results with $n^2 >> 1$ (this condition is almost always satisfied for shells of finite length).

To estimate the lowest frequency of the spectrum, it is necessary to use other theories and methods for their solution [1, 4, 12, etc.]. Other theories and solution methods have to be involved for estimating the lowest frequency of the spectrum [1, 4, 12, etc.].

The convergence of the calculations was estimated by the fundamental frequency of the vibration spectrum of the ideal ring. The results of this estimation are shown in Fig. 1.

As in Refs. [16–21], we will consider a ring making small bending vibrations in its plane and having initial

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