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Verification and adaptation of plasticity models under complex variable loading with intermediate complete and partial unloadings☆

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Abstract

The experimental studies of elasto-plastic deformation of tubular steel samples under proportional and non-proportional (monotonic and cyclic) loadings, including partial and intermediate loadings, have been conducted with the aim of improving the accuracy of the description of the complex passive loading processes. The plastic strain accumulation was observed in the course of tests carried out under passive loading. However, this effect turned out not to be described by the plastic flow theory. This result required the development of an alternative material model. The comparisons of experimental results with the predictions of the structural (rheological) elasto-plastic model and the multisurface theory of plasticity with one active surface were made. Modifications of the constitutive equations were proposed in order to improve the accuracy of the material response prediction.

Keywords: Plasticity; Passive loading; Experiment; Modeling.

Introduction

The theory of plastic flow, which has currently gained wide application, assumes that the unloading of a material has no effect on plastic strain. However, the results of a number of studies [1–19] have revealed that plastic strain accumulates under passive loading (unloading, loading within the loaded surface and tangential to it). Plastic strain under passive loading is described by the endochronic theory of plasticity [4,5] and the generalized Prandtl model (the Mazing model) [6]. The formulations of the constitutive equations of the above theories are primarily aimed at refining the description of active strain, while the parameters of the material are determined without taking into account the specifics of the processes occurring under passive loading.

Experimental studies into the properties of the plastic compliance field [20] laid the foundations for developing a multi-surface theory of plasticity with one

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active surface [14–19]. The concept of plastic compliances is directly or indirectly used by multi-surface theories of plasticity [21,22]. At the same time, not nearly enough experimental research has been accumulated on the properties of plastic compliances of materials. This mainly concerns strain with passive loading paths. The accumulation of residual strain under such loads must be taken into account not only in stress–strain analysis [14–19], but also in calculations of damage accumulation [13,23,24], studies of superplasticity processes [25] or in assessing the operating capacity and precision of machine units [26].

This study was carried out in order to further develop and substantiate the concept of plastic compliances and to devise a procedure for performing calculations related to complex variable loading involving intermediate full or partial unloadings.

Constitutive equations for describing passive loading

Vector representation of stress and strain

Under two-parameter loading with invariable and coincident directions of the principal axes of stresses and strains tensors, five-dimensional deviatoric spaces are reduced to two-dimensional ones. Various equivalent approaches are possible [7] for introducing the coordinates of a two-dimensional space that satisfy the condition that the vector length be equal to the tensor intensity. One possible way of setting the Σ_1 and Σ_2 coordinates for a point characterizing the stress state is as follows:

$$\begin{aligned}\Sigma_1 &= \sigma_i \cos\psi = \sigma_1 - \tfrac{1}{2}(\sigma_2+\sigma_3) = \tfrac{3}{2}S_1,\\ \Sigma_2 &= \sigma_i \sin\psi = \tfrac{\sqrt{3}}{2}(\sigma_2-\sigma_3) = \tfrac{\sqrt{3}}{2}(S_1+2S_2),\end{aligned} \tag{1}$$

where σ_i is the von Mises stress; ψ is the angle between the stress vector $\mathbf{\Sigma}$ and the Σ_1 axis; σ_1, σ_2, σ_3 are the principal stresses; S_1, S_2, S_3 are the principal deviatoric stresses.

Similar expressions are also introduced for plastic strains. They link the E_1^p, E_2^p coordinates of the corresponding point in the two-dimensional space of plastic strain to the principal values of deviatoric plastic strain ε_1^p and ε_2^p by the relations:

$$\begin{aligned}\mathrm{E}_1^p &= \varepsilon_i^p \cos\varphi = \varepsilon_1^p,\\ \mathrm{E}_2^p &= \varepsilon_i^p \sin\varphi = \tfrac{1}{\sqrt{3}}(\varepsilon_1^p + \varepsilon_2^p),\end{aligned} \tag{2}$$

where ε_i^p is the von Mises plastic strain intensity, φ is the angle between the plastic strain vector $\mathbf{E}^p$ and the E_1^p axis.

In the stress space, sets of stress states sharing a common attribute (the development of plastic strain of a given magnitude, fracture, etc.) correspond to certain boundaries (hypersurfaces which are, in the particular case, hyperspheres in the vector space under consideration, or curves under biaxial loading). One of these boundaries is shown in Fig. 1*b* by a circle of radius C_α with the center in point α.

Structural (rheological) model

Let us consider a structural model of plastic deformation of the material [13,15,28] (Fig. 1*a*). It is assumed within this model that the yield boundary retains its shape and size C_α (Fig. 1*b*) under loading. Its position is governed by the loading history. This boundary starts to move when the stress vector intersects it from within.

The vector of plastic strain increment $\Delta\mathbf{E}^p$ is proportional to the projection of the vector of stress increment $\Delta\mathbf{\Sigma}$ by the outer normal to the boundary of the loading surface and is directed along this normal. The lengths of the increments of the considered vectors, $|\Delta\mathbf{E}^p| = \Delta\varepsilon_i^p$ and $|\Delta\mathbf{\Sigma}| = \Delta\sigma_i$, are related by

$$|\Delta\mathbf{E}^p| = H_\alpha |\Delta\mathbf{\Sigma}| \cos\left(\Delta\mathbf{\Sigma}^\wedge \Delta\mathbf{E}^p\right), \tag{3}$$

where H_α is the plastic compliance modulus, constant for all points of the boundary.

When the stress state changes within the inner α-circle, all the elements with dry friction are fixed (Fig. 1*b*). As the boundary is reached, element *1* begins to move relative to element *2*. The stresses α_1 and α_2 emerging in the elastic elements are the coordinates of the center of the circles with the radius C_α in the plane Σ_1, Σ_2 (Fig. 1*b*). Similarly, the stresses in the elements β_1 and β_2, γ_1 and γ_2 are the coordinates of the centers of the circles with the radii C_β и C_γ. The parameters α_1 and α_2, β_1 and β_2, γ_1 and γ_2 correspond to the vectors $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

Plastic strain does not develop under loading along the *1*→*2* path (Fig. 1*b*), since the distance from the current point to the center of the circle δ_α is less than its radius C_α:

$$\delta_\alpha \doteq \sqrt{(\Sigma_1-\alpha_1)^2 + (\Sigma_2-\alpha_2)^2} \le C_\alpha. \tag{4}$$

With further motion along the straight line *1*–*2*, after point *2* ($\delta_\alpha > C_\alpha$) is passed, the element α starts to move along the radius to the α-circle, that is, at an angle ω:

$$\omega = arctg\frac{\Sigma_1-\alpha_1}{\Sigma_2-\alpha_2}, \tag{5}$$