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JID: SPJPM [m3+dc;July 5, 2017;14:18]

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St. Petersburg Polytechnical University Journal: Physics and Mathematics 000 (2017) 1–4
www.elsevier.com/locate/spjpm

A regularization of the Hartle–Hawking wave function

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Abstract

The paper puts forward a modification of the no-boundary Hartle–Hawking wave function in which, in the general case, the Euclidean functional integral can be described by an inhomogeneous universe. The regularization of this integral is achieved in arbitrary canonical calibration by abandoning integration over the lapse and shift functions. This makes it possible to 'correct' the sign of the Euclidean action corresponding to the scale factor of geometry. An additional time parameter associated with the canonical calibration condition then emerges. An additional condition for the stationary state of the wave function's phase after returning to the Lorentzian signature, serving as the quantum equivalent of the classical principle of the least action, was used to find this time parameter. We have substantiated the interpretation of the modified wave function as the amplitude of the universe's birth from 'nothing' with the additional parameter as the time of this process. A homogeneous model of the universe with a conformally invariant scalar field has been considered. In this case, two variants of the no-boundary wave function which are solutions of the Wheeler–DeWitt equation have been found.

Keywords: General relativity; Universe; Quantum cosmology; Euclidean action; Lorentzian signature; Hartle–Hawking wave function.

Introduction

The Hartle–Hawking no-boundary wave function of the universe [1,2] is a unique construction in quantum cosmology which has been put forward to describe the early stages of the universe evolution. It is possible that this function describes the whole universe evolution defining the probability measure on classical spacetimes [3]. But the problem is that, in the general case, it has been ill-defined [4] out of the scope of the semiclassical approximation, since the Euclidean action of General Relativity (GR) is not positive-definite. A negative contribution to the action is related to the conformal scale factor of geometry [5].

In the present paper we propose an adaptation (by integral regularization) of the Hartle–Hawking no-boundary wave function that allows avoiding the mentioned difficulty. Moreover, we put forward another physical interpretation of this regularization, namely, this selected state of the universe will be considered as initial, without any dynamical subject-matter. The dynamics can be formulated separately using the ordinary GR Hamiltonian with the Lorentzian signature. This is due to the fact that the proposed adaptation violates the initial covariance of the Hartle–Hawking formulation, and the obtained wave function will not generally be a solution of the Wheeler–DeWitt equation.

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http://dx.doi.org/10.1016/j.spjpm.2017.05.002

(Peer review under responsibility of St. Petersburg Polytechnic University).

Please cite this article as: N.N. Gorobey, A.S. Lukyanenko, A regularization of the Hartle–Hawking wave function, St. Petersburg Polytechnical University Journal: Physics and Mathematics (2017), http://dx.doi.org/10.1016/j.spjpm.2017.05.002

In order to determine this selected state within the Hartle–Hawking no-boundary formulation we propose to make (at our will) the change of the sign of the negative term in the Euclidean action of GR (subsequently as “Euclidean GR”).

Our first comment is that the sign will be restored afterwards. But this change is fraught with consequence: classical constraints of the Euclidean GR become unsolvable in the real range of variables’ values.

This means that integrating over the lapse and shift functions in the continual-integral representation of the no-boundary wave function becomes meaningless. Because of this, we simply fix these variables up to the next stage of our regularization procedure. In this way the Euclidean no-boundary wave function appears to be determined in a relativistic canonical calibration with a fixed Euclidean interval of time (it is arbitrary so far).

After integrating over all physical degrees of freedom, it is necessary to restore at once both the initial negative sign of the Euclidean action related to the conformal scale factor and the Lorentzian signature of the whole action by the Wick rotation of the Euclidean time in the opposite direction at the complex plane. As a result, the Euclidean no-boundary wave function will become complex. The final step of our regularization is fixation of the time parameter governing the wave function in addition.

For this purpose we propose to use the additional condition of the wave-function’s phase stationary state relative to variations of the time parameter. The condition of the phase stationary state is a quantum equivalent of the classical principal of the least action in the GR. The equations resulting from this condition fix the lapse and shift functions. Solving the stationary equations, we determine the no-boundary wave function of the universe up to a constant multiplier.

In the present paper, we consider this regularization procedure in the case of a simplest minisuperspace model of the universe with a conformal invariant scalar field. Although this example is far from an appropriate description of reality, it is suitable for its simple model [6]. In our framework the resulting regularized no-boundary wave function will be a non-trivial solution of the Wheeler–DeWitt (WDW) equation of the model, and so it will be stationary.

Minisuperspace with the conformal invariant scalar field

In the case of the homogeneous Robertson–Walker metric (Euclidean signature) which has the form

$$ds^2 = \sigma^2\left[N^2(\tau)d\tau^2 + a^2(\tau)d\Omega_3^2\right], \tag{1}$$

where $\sigma^2 \equiv (2/3\pi)m_p^2$, and $d\Omega_3^2$ is the metric of the 3D sphere with the unit radius, and the conformal invariant scalar field $\phi(\tau)$, the classical action of GR may be written in the form of Ref. [6] as follows:

$$I = I_a + I_\phi, \tag{2}$$

$$I_a = -\frac{\xi}{2}\int_0^1 d\tau\tilde{N}\left[\left(\frac{1}{\tilde{N}}\frac{da}{d\tau}\right)^2 + a^2\right], \tag{3}$$

$$I_\phi = \frac{1}{2}\int_0^1 d\tau\tilde{N}\left[\left(\frac{1}{\tilde{N}}\frac{d\psi}{d\tau}\right)^2 + \psi^2\right], \tag{4}$$

where $\tilde{N} = Na$, $\psi = \sqrt{2}\pi a\sigma\phi$.

The Wheeler–DeWitt equation of the model has the form

$$\left[\frac{1}{a^p}\frac{\partial}{\partial a}\left(a^p\frac{\partial}{\partial a}\right) - a^2 - \frac{\partial^2}{\partial\chi^2} + \chi^2\right]\Psi(a,\chi) = 0. \tag{5}$$

We have introduced a regularization parameterξ in Eq. (3) whose "normal" value is $+1$. Further, for simplicity we will take the parameter of operator ordering$p = 0$.

Following Ref. [3], let us consider the configurations of the scale factor a on a disk with boundary conditions: $a(0) = 0$(the South Pole) and $a(1) = b$ at the final spatial section.

For the initial configurations of the conformal scalar field ϕ at the South Pole let us consider two cases:

(i) $\phi(0) = 0$ ($\psi$is smooth in the South Pole);
(ii) $\dot{\phi}(0) = 0$ (ϕ is smooth in the South Pole, but $\psi(0) = 0$).

Indeed,

$$\dot{\psi}(\tau) \propto \dot{a}\phi + a\dot{\phi} = \frac{\pi}{\sqrt{2}\sigma}(l\dot{n}a)\psi + a\dot{\phi}. \tag{6}$$

It follows from here for the second case: $\psi \propto a$ in the limit.

In both cases we take $\psi(1) = \chi$ at the final spatial section. Since the integration over the (renormalized) lapse function $\tilde{N}(\tau)$ will not be performed from this point on, we obtain the dependence of the universe’s state at the final spatial section on an additional real Euclidean time parameter C:

$$C \equiv \int_0^1 d\tau\tilde{N}(\tau). \tag{7}$$