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# Hydrostatic pressure and magnetic field effect on the excited states in inverse parabolic quantum dot

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ABSTRACT

The hydrostatic pressure (P) influence of the degenerate energy states inside an inverse parabolic quantum dot (IPQD), with and without an external magnetic field, was performed within the frame of the effective mass approximation. Our theoretical results showed that the effect of relatively high pressure clearly appeared to induce a crossing between the excited states in the strong confinement region. But in the weak confinement region, such crossing disappeared and, in addition, the excited states got reordered. In the presence of an external magnetic field the hydrostatic pressure modified the crossing points of the degenerate states. We investigated the electron-heavy hole transition energy. It displayed a blue shift with increasing the pressure values and the magnetic field strength. But it showed an adhesive red shift by increasing the IPQD size.



## 1. Introduction

In recent years, the theoretical and experimental studies in nanotechnology have investigated the low dimensional quantum-structures such as semiconductor quantum well (QW), quantum well wire (QWW) and quantum dot (QD). These studies developed understanding of their optical, electronic, and magnetic properties. Away from the known widely and well-examined QW hetero-structures, which have parabolic confinement potential shape [1,2], half parabolic potential [3], graded potential [4], V-shaped potential [5], the inverse parabolic quantum well (IPQW) potential shape [6–9] have been produced and studied. The IPQW is grown by molecular-beam epitaxy, using the digital technique [10], or using the analog technique [11]. Most of the existing literature on the fundamental physical properties of the IPQW have increased lately because such confinement potential gives the possibility to realize high performance optoelectronic devices [12–14]. We believe the previous work on the inverse parabolic quantum well [IPQW] potential in low dimensional systems is very limited [7,15,16]. It is worthwhile to mention that the authors of Ref. [15], investigated the effects of the hydrostatic pressure and the external magnetic field on the donor binding energy in an inverse parabolic (IP) quantum well. Thus we may say, the previous work on IP confinement potential in QD is very rare, except the calculation of the transition energy under the magnetic field effect given by S. A. Safwan et al. [17]. The application of magnetic field and external perturbation like the hydrostatic pressure is an effective way to change the confined carrier state energy in nano-structures [18–20].

In this work, we examine the degenerate excited states of carriers under the effect of a hydrostatic pressure and an external magnetic field simultaneously, and the carriers are confined in a GaAs cylindrical quantum dot with an IPQC potential. Through, our calculations, we considered strong and weak confinement regions which are specified according to the QD dimensions. We are concerned about the perturbation which occurred due to the hump of the inverse parabolic potential. This hump is created in our examined QD by adding a percentage of Al content ($x_c$) to the dot center, such hump acts as an electrical perturbed potential inside the inverse parabolic QD [16,21]. We would like to emphasize that in such IPQD, we solved Schrodinger equation within the effective mass approximation, without considering the Γ-X mixing band because: Firstly, we examined the electronic states inside a considerable wide QD, where the smallest considered radius is $R = 10$ nm that is almost 20 times the QD-GaAs lattice constant. The Γ-X mixing band takes place in the very narrow QD, therefore the electron states localized in the barrier when the hydrostatic pressure is applied [22,23], but in the case of the three dimension IPQC potential the

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electron stays inside the IPQD at relatively high pressure. Secondly the QD structures have a small pressure coefficient than the bulk material and the QW structures [24–26].

Before we proceed, we would like to stress what makes this work unique. Firstly, we examined theoretically the electronic excited state energies of confined carriers in IPQD. Secondly, we studied such excited state energies under the effect of a hydrostatic pressure and an external magnetic field. The present paper is organized as follows: We write in section 2 our theoretical model, section 3 presents our results with a discussion. At last, we give our conclusion in section 4.

## 2. Theoretical model

Considering an electron confined in a cylindrical quantum dot with an inverse parabolic (IP) potential, under the effect of a hydrostatic pressure, or simultaneously a pressure and an external magnetic field, we investigate the electronic state energies of the charged particles (electron or hole) and their transition energy. Within the effective mass approximation the Schrodinger equation for such confined electron in IPQD is given as:

$$\frac{1}{2\ m_e^*(P)}\left(\overline{p}+\left(\frac{e}{c}\right)\overline{A}\right)^2\ \Psi(\rho,\varphi,z)+V(\rho,z,P)\Psi(\rho,\varphi,z) = E\ \Psi(\rho,\varphi,z). \quad (1)$$

here $\rho$ is its radial direction, and $\varphi$ is the electron azimuthally angle direction, respectively. The symmetric gauge within the vertical static magnetic field $\overline{B}$ is $\overline{A}=\frac{\overline{B}\times\overline{\rho}}{2}$, where $\overline{B}=(0,0,B)$, therefore, $\overline{A}=(0,A_\varphi,0)$. The confinement potential in the in-plane $V(\rho,P)$, and in the z-direction $V(z,P)$, as given in Refs. [16,17], can be written as:

$$V(\rho,P)=\begin{cases} V_i\left(1-\frac{\rho^2}{R^2}\right) & \rho\le R \\ V_0^i & \rho>R \end{cases},\ \text{and}$$
$$V(z,P)=\begin{cases} V_i\left(1-\frac{z^2}{L^2}\right) & z\le L \\ V_0^i & z>L. \end{cases} \quad (2\text{-a})$$

here R and 2L represent the radius and the height of the IPQD, respectively. $V_0^i$ is the barrier potential height and is given by: $V_0^i = Q_c\Delta E_g^\Gamma(x,\ P)$ in (eV), where $Q_c$ is the parameter of the conduction band offset. $\Delta E_g^\Gamma(x,P)$ as a function of both the pressure (P) and Al content x [20], represents the total band-gap difference between GaAs-QD material and the barrier material $Ga_{1-x}Al_xAs$ at $\Gamma$ point, and can be written as:

$$\Delta E_g^\Gamma(x,\ P)=\left(1.155\ x+0.37\ x^2\right)-P\left(1.3\times10^{-3}x\right). \quad (2\text{-b})$$

here $V_i$ is the potential strength inside a quantum dot (potential hump) induced by adding Al content $x_c$ to the dot center, let $\sigma=\frac{x_c}{x}=\frac{V_i}{V_0^i}$ [16].

According to the adiabatic approximation; The wave function $\Psi(r,\ z,\varphi)$, may be separated into two independent functions as; $\Psi(\rho,z,\varphi)=f(\rho,\varphi)g(z)$, where $f(\rho,\varphi)$ is the solution of the following in-plane equation inside the QD ($\rho\le R$). Including the confinement potential $V(\rho,P)$, we get

$$\frac{-\hbar^2}{2m_i^*(P)}\nabla_\rho^2 f(\rho,\varphi)-\frac{j\hbar\ \alpha}{2m_i^*(P)}\frac{\partial}{\partial\varphi}f(\rho,\varphi)+\left\{V_i(P)-V_i\frac{\rho^2}{R^2}+\frac{\hbar^2\alpha^2}{8\ m_i^*(P)}\rho^2\right\}f(\rho,\varphi)=E^\rho(P)f(\rho,\varphi) \quad (3\text{-a})$$

By considering, $\nabla_\rho^2=\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right)$, and $\alpha=\frac{e}{\hbar}\frac{B}{c}$.

Outside the QD ($\rho>R$), we can write,

$$-\frac{\hbar^2}{2m_e^*(P)}\nabla_\rho^2 f(\rho,\varphi)-\frac{j\hbar^2\alpha}{2m_e^*}\frac{\partial}{\partial\varphi}f(\rho,\varphi)+\left\{V_0^i+\frac{\hbar^2\alpha^2}{8\ m_e^*}\rho^2\right\}f(\rho,\varphi) = E^\rho(P)\ f(\rho,\varphi). \quad (3\text{-b})$$

Defining, $\mu_1^2=\alpha^2-\frac{2m_e^*(P)\ V_i}{\hbar^2R^2}$, $\mu_2=\frac{2m_e^*(P)}{\hbar^2}(E^\rho(P)-V_i(P))-m\ \alpha$, and $\mu_3=\frac{2m_i^*(P)}{\hbar^2}\ (V_0^i-E^\rho(P))+m\ \alpha$.

After some algebra, the solution of equation (3-a) is given as:

$$f_{inside}(\rho,\varphi)=C_1\ e^{-jm\varphi}\ \rho^{|m|}\ e^{\frac{-\mu_1\rho^2}{2}}\ \text{Hypergeometric}\ M\left(\frac{2(m+1)\ \mu_1-\mu_2}{4\mu_1},|m|+1,\mu_1\rho^2\right).$$

where $m$ represents the orbital quantum number of the system, $m=0,\ \pm1,\ \pm2,\dots$ , here M is the Hypergeometric Kummer's function. Similarly, we get the solution of equation (3-b) outside the QD as:

$$f_{outside}(\rho,\varphi)=C_2\ e^{-jm\varphi}\ \rho^{|m|}\ e^{\frac{-\alpha r^2}{4}}\ \text{Hypergeometric}\ U\left(\frac{2(m+1)\ \alpha/2\ +\mu_3}{2\ \alpha},|m|+1,\frac{\alpha\ \rho^2}{2}\right).$$

where $U$ is the confluent hypergeometric function, the normalized constants $C_1$ , and $C_2$ , determined by applying the boundary conditions.

Applying the boundary conditions to the in-plane solution when $\rho=R$, we get the following transcendental equation:

$$f_{inside}f'_{outside}-f_{outside}f'_{inside}\quad=0 \quad (4)$$

The numerical solution of equation (4) gives the state energies $E_{n,m}^\rho(P)$, where each state energy is labeled by the radial quantum number $n$ and the orbital quantum number $m$. The total energy of an electron confined in IPQD is given by:

$$E_e(P)=E_{n,m}^\rho(P)+E_{n_1}^z(P). \quad (5)$$

where $E_{n_1}^z(P)$, and $g(z)$ are the eigenvalues and the eigenstates of the Hamiltonian in the z-direction, see details in Ref. [17]. In the same procedures, we can obtain the total energy of the heavy hole considering its effective mass and its charge sign.

## 3. Results and discussion

Using the methodology for the GaAs- QD embedded in $Al_{.35}Ga_{.65}$ As, the Al concentration in the barrier material is taken as x = 0.35. We choose the conduction band offset $Q_c=0.57$ so that the confinement potential barrier are $V_0^e=0.57\Delta E_g^\Gamma(x,P)\ eV$ for the electron, $V_0^h=0.43\Delta E_g^\Gamma(x,P)\ eV$ for the heavy hole, and the

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