Journal of Crystal Growth 402 (2014) 113–118

Contents lists available at ScienceDirect

Journal of Crystal Growth

journal homepage: www.elsevier.com/locate/jcrysgro

Morphological instability of a stressed solid cylinder in the solidification and melting regimes

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ARTICLE INFO

Article history:
Received 21 March 2014
Received in revised form
19 May 2014
Accepted 20 May 2014
Communicated by P. Rudolph
Available online 27 May 2014

Keywords:
A1. Crystal morphology
A1. Growth models
A1. Morphological stability
A1. Stresses

ABSTRACT

The linear stability of a single component axi-symmetrical solid in contact with its melt is theoretically investigated with respect to the development of longitudinal sinusoidal fluctuations of wavelengths $\lambda < 2\pi R_e$, with R_e its radius, in the solidification and melting regime. In the solidification regime, the applied stress adds to the temperature gradient to favor the development of the surface fluctuations. In the melting regime where the temperature gradient has a smoothing effect, the applied stress is the main destabilizing source for the cylinder that has been found to undergo morphological instability below a critical radius.

The morphological evolution of solids is a long-standing problem that has been widely studied from both experimental and theoretical point of view. When a planar solid is submitted to uniaxial stress for example, its surface becomes unstable due to the so-called Asaro–Tiller–Grinfeld (ATG) instability [1–3]. During the solidification process of a planar solid in contact with its melt, the Mullins–Sekerka (MS) instability also takes place when the growth velocity of the solid–liquid interface exceeds a critical value [4]. The competition between ATG and MS instabilities has been then characterized and the stress has been found to modify the perturbation development [5]. In the case of a spherical particle growing in a liquid, Mullins and Sekerka [6] have demonstrated that the shape fluctuations should appear beyond a critical radius of the sphere. The effect of composition stress on the development of the surface fluctuations has been then characterized [7] in the formalism developed by Cahn and Larché [8–10] to describe the diffusion under stress in alloys. When the particle is growing into a solid matrix, the effect of misfit strain has been also studied on the morphological evolution of the particle as well as the effect of the elastic coefficients of the matrix and precipitate phases [11,12]. More recently, the effect of a controlled far-field heat flux on the evolution of two and three dimensional solids growing in a undercooled liquid has been investigated in the linear and non-linear regimes and the possibility of self-similar growth has been discussed [13–16]. Likewise, the shape evolution of a cylinder growing by diffusion in a liquid has been studied in the low supersaturation condition such that the quasi-static approximation has been used for solving the diffusion problem [17]. The cases of radial and longitudinal fluctuations have been considered. In particular, the development of longitudinal fluctuations with a wavelength increasing with the cylinder radius has been found to be favorable beyond a critical radius while perturbations of constant wavelength have been found to develop in a finite range of radius values. When the cylinder is growing in a solid phase, the development of radial fluctuations has been also characterized when a misfit strain and a supplementary stress in the matrix are considered [25,26]. The stress-induced morphological instability has been studied in the case where a solid or liquid layer is embedded in a two-phase solid under stress [28,29]. The effect of stress and elastic coefficients of the solid phases has been then characterized on the roughness development at the interfaces. The stability of a uniaxially stressed solid has been characterized with respect to the development by surface diffusion of axi-symmetrical and nonaxi-symmetrical perturbations [27].

The solidification of an ice cylinder in distilled water or in aqueous solutions has been considered from both experimental and theoretical point of view [18–20]. The development of longitudinal and radial fluctuations of the cylinder has been thus characterized in both linear and non-linear regimes and the liquid–solid interface energy has been experimentally determined. It has been found that the Mullins–Sekerka analysis [6] initially developed for a growing crystal into a binary melt is satisfied for the growing ice cylinder and that the different solutes have no measurable effect on its morphological evolution. Likewise, the mechanical behavior of a fresh-water ice cylinder under compression has been considered and the ductile-to-brittle transition and the formation of wing cracks have been characterized [21–24].

In this work, the effect of a uniaxial stress has been theoretically investigated on the development of longitudinal fluctuations

http://dx.doi.org/10.1016/j.jcrysgro.2014.05.019
0022-0248/

of the radius of a solid cylinder in contact with its melt in the solidification and melting regimes, respectively. The growth rate of the fluctuations and the critical radii associated with the morphological changes have been determined for each regime. The case of an ice cylinder in a bath of distillated water is discussed.

1. Modeling

A solid cylinder of length $2L$ and radius R_e is considered in Fig. 1 which is submitted to an external uniaxial stress σ_0 in the region $r \le R_i$ on both end-surfaces located at $z = \pm L$, with (r, θ, z) the cylindrical coordinate system. These loading conditions would correspond to a creep experiment for the cylinder under constant applied force $\pi R_i^2 \sigma_0$ when $R_i < R_e$ and when the time variation of R_i through it deformation dependence can be neglected. When $R_i = R_e$, the cylinder is thus submitted to an increasing applied force $\pi R_e^2 \sigma_0$. The cylinder is in contact with its melt such that the temperature T_b is set to be constant at $r = R_b$, where R_b is the bath radius. The stress tensor $\overline{\sigma}^{(0)}$ in the cylinder has been determined far from the two end-surfaces when its length is large compared to its radius, i.e. $2L \gg R_e$. In the framework of linear and isotropic elasticity theory, the displacement field $\mathbf{u}^0$ in the center of the cylinder has been taken to be [30]

$$u_r^0(r) = P_0 r, \tag{1}$$

$$u_z^0(z) = Q_0 z, \tag{2}$$

with P_0 and Q_0 two constants that have been determined in the infinite length approximation from the following mechanical equilibrium conditions:

$$\sigma_{rr}^0(R_e) = 0, \tag{3}$$

$$4\pi R_e^2 \sigma_{zz}^0 = 4\pi R_i^2 \sigma_0, \tag{4}$$

where the stress component σ_{ij}^0 has been determined from the Hooke law of isotropic elasticity [30] with the help of the elastic displacement field displayed in Eqs. (1) and (2). It yields

$$P_0 = -\frac{\sigma_0 \nu}{2\mu(1+\nu)} \frac{R_i^2}{R_e^2}, \tag{5}$$

$$Q_0 = \frac{\sigma_0}{2\mu(1+\nu)} \frac{R_i^2}{R_e^2}. \tag{6}$$

From Eqs. (5) and (6), the stress field in the center of the solid is given by

$$\sigma_{rr}^0(r) = \sigma_{\theta\theta}^0(r) = 0, \tag{7}$$

$$\sigma_{zz}^0 = \left(\frac{R_i}{R_e}\right)^2 \sigma_0, \tag{8}$$

where the non-zero stress component σ_{zz}^0 has been found to be constant in the cylinder of a given external radius R_e.

In the following, the kinetics of the cylinder in both solidification and melting regimes is governed by the following set of equations. It is first assumed that the temperature fields satisfy to Laplace equation [6,18–20]:

$$\nabla^2 T_L = \nabla^2 T_S = 0, \tag{9}$$

where the subscripts L and S denote the liquid and solid respectively. The boundary conditions satisfied by the temperature field in the liquid and solid phases then write at the bath and interface radii respectively labeled R_b and r_e:

$$T_L(R_b) = T_b, \tag{10}$$

$$T_L(r_e) = T_S(r_e), \tag{11}$$

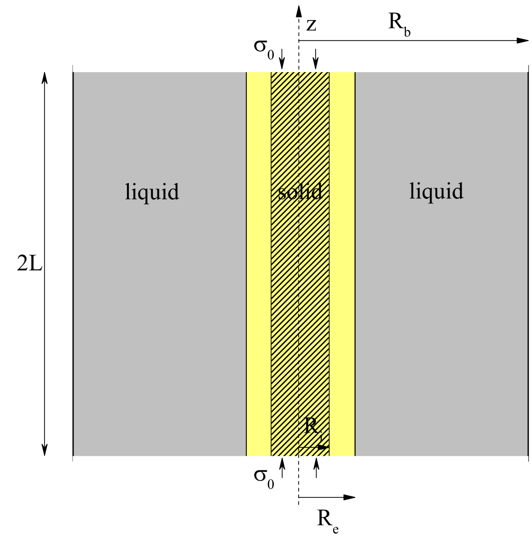

Fig. 1. A cylinder of radius R_e is considered in contact with its melt in a bath of radius R_b. A uniaxial stress σ_0 is applied at both end-surfaces on a disk of radius R_i. The length $2L$ of the cylinder is assumed to be much greater than its radius R_e.

$$T_L(r_e) = T_M - \frac{T_M}{L_V}(\gamma K + \epsilon), \tag{12}$$

with T_M is the melting temperature of a flat interface, L_V is the latent heat per unit volume of the solid, γ is the isotropic solid–liquid interface energy per unit surface and K is the interface curvature for a cylinder of radius r_e. Summation over repeated indices being implied, the elastic energy density ϵ is thus defined as

$$\epsilon = \tfrac{1}{2} \sigma_{ij} \epsilon_{ij}, \tag{13}$$

where σ_{ij} and ϵ_{ij} are the stress and strain components in the cylinder, respectively. The interface velocity V is finally determined by

$$V = L_V^{-1}(k_S \nabla T_S - k_L \nabla T_L) \cdot \mathbf{n}, \tag{14}$$

with k_L and k_S are the thermal conductivities of the liquid and solid respectively and $\mathbf{n}$ is the unit normal to the interface pointing into the liquid. It can be underlined that the diffusion-controlled growth (or melting) of the cylinder would be treated by modifying the above set of equations describing the cylinder evolution by heat flow [6]. The linear stability analysis is conducted for the cylinder whose radius equation is given by

$$r_e = R_e + \delta \cos(kr/R_e), \tag{15}$$

with δ is the perturbation amplitude and k is a positive real number that can be related to a wavelength $\lambda = 2\pi R_e / k$.

The temperature fields, stress and strain tensors have been then written as

$$T_L(r,z) = T_L^{(0)}(r) + T_L^{(1)}(r,z), \tag{16}$$

$$T_S(r,z) = T_S^{(0)} + T_S^{(1)}(r,z), \tag{17}$$

$$\sigma_{ij}(r,z) = \sigma_{ij}^{(0)}(r) + \sigma_{ij}^{(1)}(r,z), \tag{18}$$

$$\epsilon_{ij}(r,z) = \epsilon_{ij}^{(0)}(r) + \epsilon_{ij}^{(1)}(r,z), \tag{19}$$

where the indices 0 and 1 hold for the unperturbed and perturbed regimes, respectively. The general expressions of the unperturbed fields of temperature in the liquid and solid phases which satisfy to Laplace equation (9) are given by

$$T_L^{(0)}(r) = A_L^0 + B_L^0 \ln(r/R_e), \tag{20}$$