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Research articles

# Magnetohydrodynamic rotating flow and heat transfer of ferrofluid due to an exponentially permeable stretching/shrinking sheet

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ABSTRACT

This study accentuates the magnetohydrodynamic effect on three dimensional rotating flow and heat transfer of ferrofluid over an exponentially permeable stretching/shrinking sheet with suction effect. The flow and heat transfer model in partial differential equations are simplified by employing the suitable similarity transformations to a system of ordinary differential equations. Numerical results are generated by using the Matlab solver bvp4c function. The computational outcomes give significant insight into the rotating flow. The influence of three different types of base fluids are also considered, namely water, methanol and kerosene. The skin friction coefficients and the rate of heat transfer are prominently affected by the intensity of suction, magnetic field, rotating scale, concentration of nanoparticles and Prandtl number. It is found that a rise in the rotation parameter causes the ferrofluid to exert a drag force on the surface of the shrinking sheet. High intensity of the magnetic field induces higher Lorentz force and leads to the increment of the skin friction. A large concentration of nanoparticles degenerates the rate of heat transfer. On the other hand, the presence of dual solutions within the shrinking region is observed for certain values of the governing parameters. The execution of stability analysis affirms the reliability and stability of the first solution while the second solution is unstable.

## 1. Introduction

For decades, researchers have worked on the various methods to enhance the heat transfer process. Maxwell [1] initiated the idea of insertion small-sized solid particles in the base fluid to improve the thermal conductivity of the fluid. Nevertheless, there are some drawbacks of utilizing millimetre or micrometre-sized particles. Keblinski et al. [2] pointed out that the stability and rheological problems occurred when suspending particles of millimetre/micrometre-sized. Besides, some technical problems also likely to happen, for instance, particles sedimentation, clogging micro-channel of devices, high pressure drop and surface abrasion. Therefore, researchers had alternatively used smaller size of particles to overcome the deficiency. This led to the discovery of nanofluid by Choi and Eastman [3], which is a new class of fluid created from the suspension of nano-sized particles in the base fluid like water, propylene glycol, ethylene glycol, oil, etc. The synthesis of nanofluids, thermal conductivity and convective flow of nanofluids were explained in details by Das et al. [4], Buongiorno et al. [5], Shenoy et al. [6] and Nield and Bejan [7]. Further, nanofluid with magnetic or ion-based nanoparticles such as magnetite, ion, cobalt, ferric oxide and hematite is known as ferrofluid. In addition, ferrofluid is more beneficial due to its adjustable behaviour in the presence of magnetic field. Ferrofluid has captivated researchers' attention owing to the fact that there are numerous potential applications in mechano-electrical fields (computer hard drives and semiconductor), biomedical sector (curtailment of bleeding in surgeries, drug delivery, improvement of magnetic resonance image and cancer removal) and environmental engineering (energy conversion system, removal of toxic/carcinogenic materials and radioactive chemicals) [8–11]. Experimental studies of ferrofluid with consideration of various aspects were done by many researchers such as Ghofrani et al. [12], Patel et al. [13] and Yang et al. [14]. Habera and Hron [15] implemented magnetic force into the incompressible Navier-Stokes equation to solve the free-surface ferrofluid model. The influence of thermal radiation and partial slip of ferrofluid flow was investigated by Rashad [16]. He found that the promotion of slip factor contributed to the big decrement of the skin friction coefficients while the increment of thermal radiation increased the Nusselt number. Gibanov et al. [17] studied the impact of uniform inclined magnetic field on mixed convection heat transfer of ferrofluid and discovered that high intensity of magnetic field or a large value of

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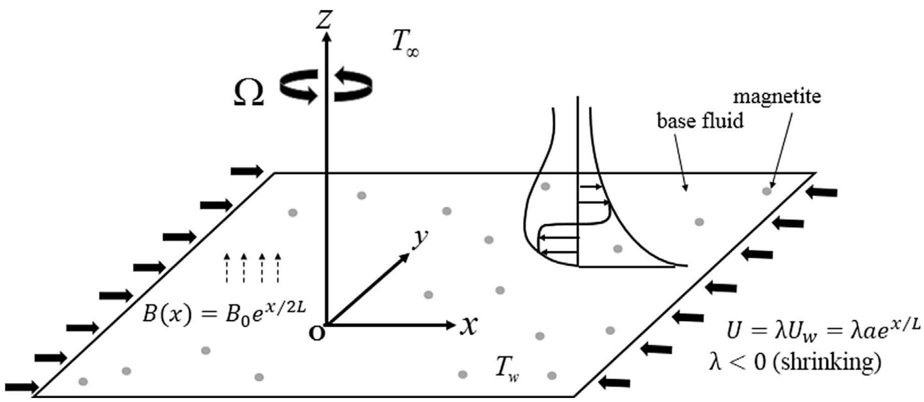


**Fig. 1.** Physical model and coordinate system.

Hartmann number reduced the heat transfer rate.

Theoretical and experimental studies of ferrofluid flow and heat transfer are highly related to the magnetohydrodynamic (MHD) concept. Alfvén [18] pioneered the works on MHD and hypothesized that if a magnetic field was constantly imposed to a conducting liquid, then the movement of the liquid induced the electromagnetic force and produced electric currents which in turn changed the state of motion of the liquid. Gul et al. [19] elaborated that the polarization of magnetite ($Fe_3O_4$) in the magnetic field direction was very strong and it kept polarized even the magnetic field was abolished. Stimulus in the research of MHD stems from its enormous applications in petroleum industries, geophysical problem, and agricultural engineering. Besides, the interaction of electrically conducting liquid and magnetic field strongly affects industrial equipment like bearings, MHD generators and pumps. MHD concept also can be utilized in medical sector such as tumors elimination, gastric medications, wound treatment and sterilization of devices [20–22]. Tashtoush and Magableh [23] studied the blood flow in multi-stenosis arteries and found that the magnetic field could modify the patterns of the flow. The effect of magnetic field on the convection of nanofluid also has been investigated for various conditions and circumstances by using the Galerkin method [24–27]. Mukhopadhyay [28] investigated MHD boundary layer flow with the presence of thermal stratification and concluded that the higher magnetic parameter suppressed the velocity field and subsequently enhanced the skin friction coefficient. The same result was obtained by Tian et al. [29] where they explored the effects of radiation optical properties and Lorentz force on MHD boundary layer flow over a stretching plate. They explained that the fluid flow was suppressed by the magnetic force significantly, and hence the convective heat transfer was terminated effectively.

Currently, researchers have a lot of interest in the investigation of boundary layer flow and heat transfer over a stretching/shrinking sheet. This is due to its remarkable significance in manufacturing and engineering processes such as extraction of polymer and rubber sheet, annealing and tinning of copper wires, glass–fiber production, crystal growing, hot rolling and drawing of continuous filaments through quiescent fluid [30–32]. Implication of slip boundary conditions in the model of MHD stagnation point flow towards a stretching/shrinking sheet was presented by Aman et al. [33] where the thermal slip reduced the temperature gradient at the surface. Dash et al. [34] identified a decrement in the skin friction with the acceleration of shrinking velocity in which defeated the effect of magnetic field. Mahmood et al. [35] implemented Chebyshev Spectral Newton Iterative Scheme to solve hydromagnetic flow over a nonlinearly stretching/shrinking sheet and discovered dual solutions in the shrinking range. Different trends of velocity and temperature profiles are observed by Thumma et al. [36] between stretching and shrinking sheets with a rise in Richardson number, Eckert number and Prandtl number. Thus, the aforementioned results proved that the stretching/shrinking surface has a noteworthy effect on the fluid flow and heat transfer.

Furthermore, another topic of interest among the researchers is to study the fluid flow and heat transfer in a rotating system. The concept of rotating flow is utilized in astrophysics, fluid engineering and geophysics. For example, in rotational air cleaners, centrifugal pumps, gas-turbine engineering and rotor-stator system. It ought to be noticed that there are some papers on the mathematical and numerical modelling of rotating flow had been published since years ago. Yadav et al. [37] studied the onset of double-diffusive nanofluid convection in a rotating porous medium layer with thermal conductivity and viscosity variation. Zin et al. [38] provided the analytical solution of MHD free convection flow of rotating Jeffrey nanofluid by utilizing the Laplace transform method. An upsurge in rotating parameter lessened the boundary layer thickness as observed by Javed et al. [39] and Zaimi et al. [40]. In addition, rotation exhibited the stabilizing effect and delayed the onset of nanofluid convection [41,42]. Further, Mustafa [43] presented the effect of partial slip on MHD nanofluid flow over a rotating disk where the magnetic field strength and velocity slip coefficient reduced the radial, axial and azimuthal velocities.

The above studies gave ideas and motivated the authors to explore deeper on the fluid flow and heat transfer in the rotating system. To the best knowledge of the authors, MHD rotating flow and heat transfer of ferrofluid due to an exponentially stretching/shrinking sheet has not been studied before. Therefore, the authors were initiated to investigate this problem by implementing Tiwari and Das model [44]. This study is extended from Mushtaq et al. [45] with the consideration of ferrofluid, shrinking sheet, magnetohydrodynamic and suction effect. Outstanding numerical results are generated by using bvp4c function in Matlab and dual solutions have been discovered. Finally, stability analysis is conducted to determine the reliability of the solutions.

## 2. Mathematical formulation

A rotating flow of three-dimensional magnetohydrodynamic ferrofluid subjected to the exponentially stretching/shrinking surface is considered with the inclusion of the mass suction. The coordinate system and physical model are illustrated in Fig. 1 where the ferrofluid flow is considered in the space $z \geqslant 0$ and the stretching/shrinking surface is aligned with the $xy$-plane. The surface is exponentially in $x$-direction with the velocity $U_w(x, y) = ae^{x/L}$. In addition, the ferrofluid is subjected to the uniform rotation about z-axis with constant angular velocity $\Omega$. The thermophysical properties of magnetite ($Fe_3O_4$) and different types of base fluid (water, methanol and kerosene) are listed in Table 1. The fluid is assumed to be electrically conducted with the application of magnetic field $B(x) = B_0e^{x/2L}$ orthogonal to the $xy$-plane. The physical model is interpreted by the mathematical governing equations with assumptions that the ferrofluid is laminar, incompressible and Newtonian. The nanoparticle and fluid phases are assumed to be in the thermal equilibrium state. The governing