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Research articles

Effects of iron nanoparticles' shape on convective flow of ferrofluid under highly oscillating magnetic field over stretchable rotating disk

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ABSTRACT

The persistence of the current article is to discuss the iron nanoparticles' shape in flows due to highly oscillating magnetic field over a stretchable rotating disk. For ferrofluid, water is considered as base fluid with suspension of iron nanoparticles having sphere, oblate ellipsoid and prolate ellipsoid shapes with different sizes. The impact of the nanoparticles' shape on velocity and temperature profiles, convective heat transfer coefficient, radial and transverse shear stress is deliberated through graphs and tables. The presence of highly oscillating magnetic field forces the particles to rotate faster than the fluid and, as a result, the total viscosity is certainly reduced. The governing equations, which are firstly modeled and thereafter converted into nonlinear ordinary differential equations in dimensionless form using similarity approach, are analytically solved using the Mathematica package BVPh 2.0 which is based on the homotopy analysis method (HAM).

1. Introduction

Ferrofluids consist of a carrier fluid loaded with small (nanometer sized) magnetic particles. The behavior of these fluids varies due to the carrier fluid, temperature, particle size, shape and loading, magnetic characteristics of the particles and the applied magnetic field. Lately, many studies have been done to investigate the characteristic of ferrofluid as function of particle volume fraction and magnetic field strength, but these do not depend on particle size and shape effect [1–10]. Although the relationship amongst shape and magnetization is not as straight, the impact of different geometries on magnetic properties keeps on being assessed. De Vicente et al. [11] prepared magnetite rod-like particles with average diameter and length of 560 nm and 6.9 μm, respectively. These works showed that magneto-rheological performance is significantly improved for elongated magnetic particles under small-amplitude shear and simple steady shear flows, hence suggesting that particle shape strongly affects the structuration under an external field. In another study [12], magneto-rheological performances of magneto-rheological fluids was investigated by using iron particles of different shapes like spherical, plate-like, and rod-like and found better improvement by non-spherical particles. De Gans et al. [13] investigated the influence of particle size on the magneto rheological properties of an inverse ferrofluid. For small particles, a strong increase of magneto rheological properties was found. In view of heat transfer, Ellahi et al. [14] discussed the particle shape effects on heat transfer rate and nanofluid flow. Their fallouts show that heat transfer rate can be improved through taking different shapes of particles. In another study, R. Ellahi et al. [15] instigated theoretical study on ferrofluid by taking spherical nanoparticles. They found 7.86% heat-transfer enhancement in the absence of a magnetic field and found 8.73% heat-transfer enhancement in present of a magnetic field. J. Fang et al. [16] used ellipsoids magnetic nanoparticles in their study, possessed remarkably enhanced thermal stability for maintaining tiny particle sizes and excellent dispersibility even under higher temperature. Effect of space dependent magnetic field on ferrofluid flow and heat transfer is investigated by M. Sheikholeslami and M. M. Rashidi [17]. They found that heat transfer coefficient is increased as increasing of magnetic number and nanoparticle volume fraction. A numerical study on the heat transfer of ferrofluids in microchannels was conducted by Xuan et al. [18]. They finally concluded that heat-transfer rate could increase if the directions of magnetic field gradient and fluid flow are the same. It is remarkable that there are still only relatively few such publications. To apply the ferrofluid to practical heat transfer processes, more studies on its flow and heat transfer feature are needed.

To the best our knowledge, a theoretically study on the effect of particle shape on characteristics of ferrofluid flow, a study in which only shape changes while the rest of the parameters are kept practically constant, is missing in the literature. In present study, effects of particle

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Nomenclature

$\mathbf{V}$	Velocity
v_r, v_θ, v_z	Velocity components
k	Thermal conductivity
r, θ, z	Cylindrical coordinates
T	Temperature of fluid
$\boldsymbol{M}$	Magnetization of fluid
$\boldsymbol{H}$	Strength magnetic field
υ	Kinematic viscosity
H_o	Amplitude of the field
τ_s	Relaxation time parameter
τ_B	Brownian relaxation time
μ_0	Permeability of free space,
β	Thermal expansion coefficient
k_B	Boltzmann constant
m	particle Magnetic moment
n	Number of particles
Ω	Vorticity of the flow
$\mathbf{M}_o$	Equilibrium magnetization
ω_o	The angular frequency of the applied magnetic field
$\omega_{\mathbf{p}}$	Internal angular momentum of particles
ξ	The ratio of magnetization energy
$\boldsymbol{I}$	Moments of inertia of the particles
p	Pressure
α	Phase angle
t	Time
ρ	Density
μ	Viscosity
C_p	Specific heat
χ'	Magnetic susceptibility

Subscripts

c	Backbone particles
int	Cluster
nf	Composition of particles and fluid
s	Solid particle
nc	Dead ends particles
a	Aggregation
f	Base fluid

shapes, particle size and high oscillating magnetic field on fluid flow and heat transfer over rotating stretchable disk are demonstrated. To achieve this goal, the present work is organized in the following way. In mathematical formulation section, a controllable force is introduced into the fundamental hydrodynamic equations which produced as negative viscosity effects. These equations are transformed into ordinary differential equations by applying appropriate transformations. In addition, the correlation models of physical properties for spherical and non-spherical particles are also discussed in this section. The solution for problem and the accuracy of method is discussed in solution of the problem section. The impact of pertinent flow quantities on velocity and temperature profiles as well as on convective heat transfer coefficient are demonstrated and discussed in results and discussion section. In last section, achievements of study are concluded and a way to enhance the convective heat transfer in fluid flow is given.

2. Mathematical modeling of the problem

2.1. Flow modeling

Consider the axially symmetric laminar and non-conducting flow of an incompressible nano-Ferrofluid past a stretchable rotating disk that has an angular velocity varying with time $\Omega_v/(1-\beta_v t)$. The coordinate system and geometry of the problem are shown in Fig. 1. We consider that the disk rotation speed has a form of $\Omega_v r/(1-\beta_v t)$ and the disk stretching velocity is $\alpha_v\,\Omega_v\,r/(1-\beta_v t)$, which is proportional to the radius r.

The basic governing equations containing continuity, motion, temperature, magnetization and rotational motion equations in vector form are [8,19]

$$\nabla\cdot\mathbf{V} = 0, \tag{1}$$

$$\rho_{nf}\frac{d\mathbf{V}}{dt} = -\nabla p + \mu_{nf}\nabla^2\mathbf{V} + \mu_0(\boldsymbol{M}\cdot\nabla)\boldsymbol{H} + \frac{\boldsymbol{I}}{2\tau_s}\nabla\times(\omega_{\mathbf{p}}-\Omega), \tag{2}$$

$$\boldsymbol{I}\frac{d\omega_{\mathbf{p}}}{dt} = (\boldsymbol{M}\times\boldsymbol{H}) - \frac{\boldsymbol{I}}{\tau_S}(\omega_{\mathbf{p}}-\Omega), \tag{3}$$

$$\frac{d\boldsymbol{M}}{dt} = \omega_{\mathbf{p}}\times\boldsymbol{M} - \frac{1}{\tau_B}(\boldsymbol{M}-\mathbf{M}_0), \tag{4}$$

$$(\rho C_p)_{nf}\frac{dT}{dt} = k_{nf}\nabla^2 T. \tag{5}$$

In above, $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla$, $\mathbf{V} = (v_r, v_\theta, v_z)$ is velocity, T is temperature, $\boldsymbol{M}$ is magnetization of the fluid, $\boldsymbol{H}$ is strength magnetic field, τ_s is Relaxation time parameter, τ_B is Brownian relaxation time, μ_0 is permeability of free space, $\boldsymbol{I}$ is sum of moments of inertia of the particles per unit volume, $\omega_{\mathbf{p}}$ is internal angular momentum due to the self-rotation of particles and Ω is the vorticity of the flow. Mean angular velocity of the particle

The complete set of equations also includes the Maxwell's equations

$$\nabla\times\mathbf{H} = 0,\ \nabla\cdot\mathbf{B} = 0, \tag{6}$$

where $\boldsymbol{B} = \mu_0(\boldsymbol{H}+\boldsymbol{M})$.

Instantaneous equilibrium magnetization $\mathbf{M}_o$ at $\tau_B = 0$ in terms of the Langevin function is defined as

$$\boldsymbol{M}_o = nmL(\xi)\frac{\boldsymbol{H}}{H},\ \xi = \frac{mH(t)}{k_B T_a},\ L(\xi) = \coth\xi - \frac{1}{\xi}. \tag{7}$$

In above, magnetic moment of the particles and number of particles are denoted by m and n respectively. Langevin function is $L(\xi)$ in which ξ is the ratio of magnetization energy. Boltzmann constant and absolute temperature are denoted with k_B and T_a symbols.

Here, the inertial term is as small as compared to relaxation term $\boldsymbol{I}\frac{d\omega_{\mathbf{p}}}{dt} \ll \boldsymbol{I}\frac{\omega_{\mathbf{p}}}{\tau_s}$. So, Eq. (3) can be rewritten as

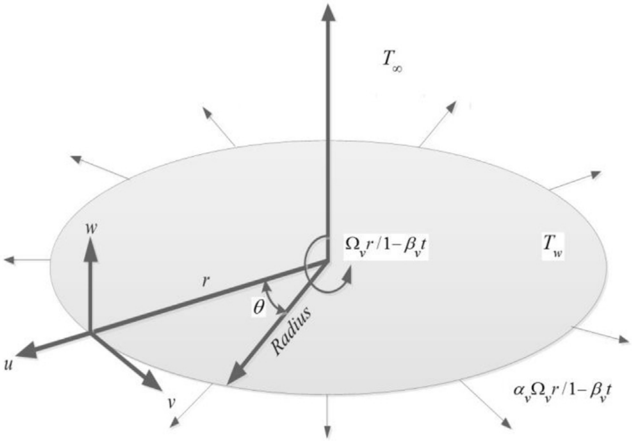

Fig. 1. Geometry of the problem.