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Phase diagram of the $O(n)$ model with defects of "random local anisotropy" type

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ABSTRACT

It is shown that the Imry-Ma theorem stating that in space dimensions $d < 4$ the introduction of an arbitrarily small concentration of defects of the "random local anisotropy" type in a system with continuous symmetry of the n-component vector order parameter ($O(n)$ model) leads to the long-range order collapse and to occurrence of a disordered state, is not true if an anisotropic distribution of the defect-induced random easy axes directions in the order parameter space creates a global anisotropy of the "easy axis" type. For a weakly anisotropic distribution of the easy axes, in space dimensions $2 \leq d < 4$ there exists some critical defect concentration, when exceeded, the inhomogeneous Imry-Ma state can exist as an equilibrium one. At lower defect concentration the long-range order takes place in the system. For a strongly anisotropic distribution of the easy axes, the Imry-Ma state is suppressed completely and the long-range order state takes place at any defect concentration.

1. Introduction

In their classical paper [1] Imry and Ma arrived at the conclusion that in space dimensions $d < 4$ the introduction of an arbitrarily small concentration of impurities of the "random local field" type into a system with continuous symmetry of the n-component vector order parameter ($O(n)$-model) leads to the long-range order collapse and to the occurrence of a disordered state, which in what follows will be designated as the Imry-Ma state. Later on their arguments were extended to impurities of the "random local anisotropy" type [2,3].

On a consideration of the amorphous magnetic with similar impurities near a phase transition [4] the statement was formulated that the disorder isotropy is an exacting requirement for the Imry-Ma disordered state occurrence.

The goal of the present paper is formulating the conditions for the occurrence of the Imry-Ma disordered state depending on the degree of anisotropy of the distribution of local anisotropy easy axes directions in the order parameter space.

2. Energy of a system of classical spins

The exchange-interaction energy of n-component localized unit (a vector length can be included to corresponding interaction or field constants) spins $\mathbf{s}_i$ comprising the simple cubic d-dimensional lattice, within the nearest neighbors approximation, has the form

$$W_{ex} = -\frac{1}{2}J\sum_{i,\delta}\mathbf{s}_i\mathbf{s}_{i+\delta}, \tag{1}$$

where J is the exchange interaction constant, the summation in i is performed over the whole spin lattice, and the summation in δ is performed over the nearest neighbors.

The energy of interaction between the spins and "random local anisotropy" type defects is

$$W_{def} = -\frac{1}{2}K_0\sum_{l}(\mathbf{s}_l\mathbf{n}_l)^2. \tag{2}$$

Here $K_0 > 0$ is the random anisotropy constant, the summation is performed over defects randomly located in the lattice sites, and $\mathbf{n}_l$ is a unit vector prescribing the random easy axis direction.

Turning to the continuous distribution of the order parameter $\mathbf{s}(\mathbf{r})$, let the inhomogeneous exchange energy be introduced in the form [5]

$$W_{ex} = \frac{D}{2}\int d^d\mathbf{r}\frac{\partial\mathbf{s}^{\perp}}{\partial x_i}\frac{\partial\mathbf{s}^{\perp}}{\partial x_i}, \tag{3}$$

where $D = Jb^{2-d}$, b is the interstitial distance, and $\mathbf{s}^{\perp}(\mathbf{r})$ is the order parameter component orthogonal to its mean direction $\mathbf{s}_0$.

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3. Imry and Ma arguments

Let us reproduce the arguments by Imry and Ma [1] as applied to impurities of the "random local anisotropy" type [6]. In the case of substantially non-collinear distribution of local easy axes directions in the order parameter space, the impurities with a particular direction of their easy axes dominate in a system volume with some characteristic dimension L due to concentration fluctuations, and thus a certain averaged anisotropy arises, the corresponding constant is

$$K_L \sim K_0\sqrt{x}\left(\frac{b}{L}\right)^{d/2}, \tag{4}$$

where x is the dimensionless concentration of impurities (the number of impurities per a unit cell). If the order parameter vector follows the space fluctuations of the easy axis directions, then it takes place a gain in the volume anisotropy energy density comparing to a homogeneous state. The addition to the volume energy density comprises the quantity of the following order

$$w_{fl} \sim \frac{-K_L}{b^d} \sim -K_0\sqrt{x}(bL)^{-d/2} \propto L^{-d/2}. \tag{5}$$

Due to a subsequent inhomogeneity in the order parameter the volume density of the exchange energy increases by the value

$$w_{ex} \sim \frac{J}{b^{d-2}L^2} \propto L^{-2}. \tag{6}$$

Hence for the space dimension $d < 4$ the long-wave (corresponding to big L) fluctuations of the order parameter direction become energetically favorable and the Imry-Ma disordered state arises. The optimum size L^* corresponding to a minimum overall energy density $w = w_{fl} + w_{ex}$ equals

$$L^* \sim b\left(\frac{J^2}{xK_0^2}\right)^{\frac{1}{4-d}}. \tag{7}$$

For the Imry-Ma state the addition to the volume density of the ordered state energy is

$$w \sim -\frac{K_0}{b^d}x^{\frac{2}{4-d}}\left(\frac{K_0}{J}\right)^{\frac{d}{4-d}}. \tag{8}$$

In the case of perfectly isotropic distribution of easy axes directions in the order parameter space, the arguments presented above are not objectionable.

4. Global anisotropy

The anisotropy contribution to the volume energy density linear in the anisotropy constant K_0 is

$$w^{(1)}(\mathbf{s}_0) = -\frac{xK_0}{2b^d}\langle(\mathbf{s}_0\mathbf{n}_l)^2\rangle, \tag{9}$$

where the brackets < > denote averaging over the whole set of defects. If the global anisotropy to the first order in K_0 is absent, that is subject to the condition $w^{(1)}(\mathbf{s}_0) = const$, one should take account of the order parameter inhomogeneity induced by random anisotropy and calculate the energy contribution quadratic (or higher power) of the constant K_0. Actually the expansion is performed in terms of a small parameter K_0/J. We neglect the longitudinal susceptibility of the system at low temperatures, much smaller than the temperature of magnetic ordering.

The presence of random anisotropy leads to a local deviation of the order parameter from its mean value and to the appearance of the component $\mathbf{s}^{\perp}(\mathbf{r})$ orthogonal to $\mathbf{s}_0$. The order parameter to the linear in $\mathbf{s}^{\perp}(\mathbf{r})$ approximation can be represented as

$$\mathbf{s}(\mathbf{r}) = \mathbf{s}_0 + \mathbf{s}^{\perp}(\mathbf{r}) \tag{10}$$

where $|\mathbf{s}_0| \leqslant |\mathbf{s}^{\perp}(\mathbf{r})|$. By substituting this expression to Eq. (2), we obtain linear in $\mathbf{s}^{\perp}(\mathbf{r})$ and quadratic in K_0 (as we shall see subsequently) summand to W_{def}

$$W_{def}^{(2)} = -b^{-d}\int d^d\mathbf{r}K(\mathbf{r})(\mathbf{s}_0\mathbf{n}(\mathbf{r}))(\mathbf{s}^{\perp}(\mathbf{r})\mathbf{n}(\mathbf{r})), \tag{11}$$

where $K(\mathbf{r}) = K_0 b^d\sum_l\delta(\mathbf{r}-\mathbf{r}_l)$ and $\mathbf{r}_l$ is the defect radius-vector. The quantity $K(\mathbf{r})(\mathbf{n}(\mathbf{r})\mathbf{s}_0)\mathbf{n}(\mathbf{r})$ plays the role of an effective random field that acts on a spin. This field component $\mathbf{h}_{eff}^{\perp}(\mathbf{r})$ orthogonal to $\mathbf{s}_0$ is

$$\mathbf{h}_{eff}^{\perp}(\mathbf{r}) = K(\mathbf{r})(\mathbf{s}_0\mathbf{n}(\mathbf{r}))[\mathbf{n}(\mathbf{r}) - \mathbf{s}_0(\mathbf{s}_0\mathbf{n}(\mathbf{r}))]. \tag{12}$$

The Fourier component $\mathbf{s}^{\perp}(\mathbf{k})$ is related to the Fourier component of the effective random field $\mathbf{h}_{eff}^{\perp}(\mathbf{k})$

$$\mathbf{s}^{\perp}(\mathbf{k}) = \chi^{\perp}(\mathbf{k})\mathbf{h}_{eff}^{\perp}(\mathbf{k}) \tag{13}$$

where $\chi^{\perp}(\mathbf{k})$ is the Fourier component of the corresponding susceptibility of the spin system.

Substitution of the expression for $\mathbf{s}^{\perp}(\mathbf{r})$ into Eq. (11) gives the quadratic in K_0 contribution to volume density of the interaction energy

$$w_{def}^{(2)} = -xK_0^2\tilde{\chi}^{\perp}[\langle(\mathbf{s}_0\mathbf{n}_l)^2\rangle - \langle(\mathbf{s}_0\mathbf{n}_l)^4\rangle] \tag{14}$$

where

$$\tilde{\chi}^{\perp} = \int\frac{d^d\mathbf{k}}{(2\pi)^d}\chi^{\perp}(\mathbf{k}), \tag{15}$$

and summation over $\mathbf{k}$ is performed over the Brillouin zone

If $w^{(1)}(\mathbf{s}_0) = const$, then the second summand in the right-hand side of Eq. (14) describes the global anisotropy of the system.

The quadratic in K_0 contribution of the inhomogeneous exchange energy (3) to the volume energy density can be found in the similar way, by substituting the expression for $\mathbf{s}^{\perp}(\mathbf{r})$ into Eq. (3). Such a contribution appears to be of the opposite sign and one half in magnitude as to $w_{def}^{(2)}$ given by Eq. (14). Therefore the resulting volume density of the anisotropy energy takes the form

$$w^{(2)} = -\frac{x\tilde{\chi}^{\perp}K_0^2}{2}[\langle(\mathbf{s}_0\mathbf{n}_l)^2\rangle - \langle(\mathbf{s}_0\mathbf{n}_l)^4\rangle] \tag{16}$$

The following value is taken as a global anisotropy constant K_{eff}

$$K_{eff} = 2b^d\left[(w^{(1)} + w^{(2)})_{max} - (w^{(1)} + w^{(2)})_{min}\right], \tag{17}$$

where w_{max} and w_{min} are maximum and minimum values of $w^{(1)} + w^{(2)}$ as a function of vector $\mathbf{s}_0$ direction.

For space dimensions $2 < d < 4$, one can use the susceptibility of the pure system for the quantity $\chi^{\perp}(\mathbf{k})$ and the quantity $\tilde{\chi}^{\perp}$ has no peculiarities at $\mathbf{k} = 0$. A specific feature of two-dimensional models is the absence of the long-range order in a pure system at finite temperature, and so one has to anticipate the existence of the long-range order induced by random local anisotropy axes and solve a self-consistent problem [7].

Since under the influence of random field the order parameter deviates from the easy direction to the hard one, the expression for $\chi^{\perp}(\mathbf{k})$ takes the form

$$\chi^{\perp}(\mathbf{k}) = \left(Jb^2k^2 + K_{eff}\right)^{-1}. \tag{18}$$

It can be easily seen that K_{eff} cuts the divergence of $\tilde{\chi}^{\perp}$ at small $\mathbf{k}$ values which bring the main contribution to $\tilde{\chi}^{\perp}$ for $d = 2$. As the result we obtain

$$\tilde{\chi}^{\perp} = \frac{1}{4\pi b^2 J}\ln\frac{4\pi J}{K_{eff}}. \tag{19}$$

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