Journal of Magnetism and Magnetic Materials 458 (2018) 213–216

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Massive photons in magnetic materials from nonlocal quantization

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ARTICLE INFO

Article history:
Received 20 January 2018
Received in revised form 6 March 2018
Accepted 7 March 2018
Available online 13 March 2018

Keywords:
Nonlocal-in-time kinetic energy
Acceleratum operator
Massive photons in magnetic materials
Modified Landau energy and levels positions

ABSTRACT

In this letter, we have discussed the implications of nonlocal-in-time kinetic energy approach recently introduced by Suykens on the dynamics of charge particles spin in a magnetic field with spin. It was observed that both the Landau quantization of the energy and their levels positions are modified. Besides, massive photons are generated in magnetic materials similar to the one encountered in massive quantum electrodynamics. These massive photons depend on the electron mass and on the nonlocal relaxation time. For solid copper, we found $M \approx 3.4 \times 10^{-20}\ \alpha^{-2}\ eV/c^2$ where α is a real free parameter.

There exists currently an ingoing growing interest in higher-derivative models in classical and quantum field theory, either from the applicable point of view or from the fundamental one. In the past, these theories were avoided because of undesirable properties related to states of negative norms. However, due to their generic features, higher-order derivative theories constitute a motivating challenge in sciences. These theories are characterized by the presence of an infinite number of the higher-order temporal derivatives of the coordinates in the Lagrange function and they don't disagree with the formalism of quantum theory [1–5]. In classical theories, most of the dynamical equations are governed by 2nd-order differential equations. However, there are some exceptions to this rule and the most well-known example is the Abraham-Lorentz theory which describes the equation of motion for charged particles taking into account radiative effects [6]. There exist several methodologies to deal with higher-order derivative theories, e.g. the method of perturbative constraints introduced in [7] which is used in higher-order dynamical systems that are "truncated perturbative expansions of nonlocal dynamical systems" where their equations of motion depend on more than one moment in time. These nonlocal theories are observed in electrodynamics where charged particles interact by means of retarded potentials, i.e. the forces acted on a particle depend on the history of both its proper position and the rest of particles positions. A more recent interesting approach to deal with higher-order derivatives in the one introduced by Suykens in [8] which in fact is motivated from Feynman's observation of the kinetic energy functional which can be written as $\frac{1}{2}m\frac{x_{k+1}-x_k}{\varepsilon}\frac{x_k-x_{k-1}}{\varepsilon}$ in place of $\frac{1}{2}mvv$ with $\varepsilon = t_{i+1} - t_i$, i.e. particle positions are shifted backward and forward in time (m being the mass of the body and $v = \frac{dx}{dt} \equiv \dot{x}$ its velocity). Suykens in contrast use the shifting-coordinates methodology and rewrite the kinetic energy as $K = \frac{1}{2}mv\frac{\Delta}{2}$ where $\Delta = \dot{x}(t+\tau) + \dot{x}(t-\tau)$ and τ is a somewhat tiny parameter entitled the "nonlocal time parameter". This simple maneuver leads to a number of motivating properties at large and small scales which were discussed in a series of research papers [9–15]. Obviously, nonlocal terms expanded in the following Taylor series $x(t+\tau) = x(t) + \sum_{k=1}^{n}\frac{\tau^k}{k!}x^{(k)}(t)$ and $x(t-\tau) = x(t) + \sum_{k=1}^{n}\frac{(-\tau)^k}{k!}x^{(k)}(t)$ became higher-derivative expansions. Due to the relevance of nonlocality in classical electrodynamics [6,16], in Weyl semi-metals [17], in superconducting films [18], in linearly accelerated systems [19], in superconductor [20], in planar Josephson junctions [21], in arrays of quantum dots [22] among others, we discuss in this letter the quantizing dynamics inside a solid of a charged particle in the presence of a magnetic field with spin. Nevertheless, one of the main consequences of Suykens's nonlocal-in-time kinetic energy approach is the emergence of an acceleratum operator $\hat{a} = \frac{\hat{p}c}{\alpha i m}\nabla$ based on Caianiello's maximal acceleration arguments in quantum mechanics [13]. Here $\hat{p} = -i\hbar\nabla$ is the quantum momentum operator, c the celerity of light, $i = \sqrt{-1} \in \mathbb{C}$ and α is a real parameter. Since we are considering an upper limit for the acceleration of theparticle, the third-order of the position (the jerk) is neglected. It will be of interest to explore the consequences of its presence in the nonlocal electrodynamics theory with spin.

In the absence of the spin, the motion of a charged particle in a magnetic field has the Lagrangian $L = \frac{1}{2}m\vec{v}^2 - q\phi + q\vec{v}\cdot\vec{A}$, q being the charge, ϕ the scalar potential and $\vec{A}$ the vector potential, e.g.

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https://doi.org/10.1016/j.jmmm.2018.03.012
0304-8853/

electrons in a metal. Replacing the kinetic energy term by $K=\frac{1}{2}mv_{\frac{\Delta}{2}}^{2}$ and performing the Taylor series expansions of $\Delta=\dot{\vec{x}}(t+\tau)+\dot{\vec{x}}(t-\tau)$ about $\tau=0$, we find:

$$L_{\tau,n}=\frac{1}{2}m\vec{v}^2+\frac{1}{4}m\vec{v}\left(\sum_{k=1}^{n}\left(1+(-1)^k\right)\frac{\tau^k}{k!}\vec{x}^{(k+1)}\right)-q\phi+q\vec{v}\cdot\vec{A}, \quad (1)$$

and the corresponding Hamiltonian is given by:

$$H_{\tau,n}=\sum_{k=0}^{n+1}\sum_{j=0}^{k-1}(-1)^j\left(\vec{x}^{(k-j)}\frac{d^j}{dt^j}\frac{\partial L_{\tau,n}}{\partial\vec{x}^{(k)}}\right)-L_{\tau,n}. \quad (2)$$

For $n>2$, terms on $\tau^4,\tau^6,\ldots$ occur in Eq. (1) which could be neglected. Therefore we limit our analysis up to $n=2$ and since we wish to take into account the acceleratum operator, we neglect the term $\vec{x}^{(3)}$ in Eq. (2) and we find after algebra the Hamiltonian momentum operator

$$\hat{H}_{\tau,2}=\frac{1}{2m}\left(\hat{\vec{p}}-q\vec{A}\right)^2-\frac{1}{4}\tau^2m\hat{\vec{a}}^2+q\phi+O(\tau^3). \quad (3)$$

In the presence of a single spin, it is required to add to Eq. (3) the Hamiltonian $\hat{H}=-\frac{2g\mu_B}{\hbar}\hat{S}$ where $\mu_B=\frac{q\hbar}{2m}$ is the Bohr magnetron, g is the electron-spin g-factor and $\hat{S}$ is the spin operator. In particular for the case of a constant magnetic field pointing in the z-direction, i.e. $\vec{B}=(0,0,B_z)$ and $\phi=0$ we find:

$$\hat{H}_{\tau,2}=\underbrace{\frac{1}{2m}\left(\left(\hat{\vec{p}}-q\vec{A}\right)^2-\frac{1}{2}\tau^2m^2\hat{\vec{a}}^2\right)}_{\hat{H}_{1,\tau}}\underbrace{-\frac{2g\mu_B}{\hbar}B_z\hat{S}_z}_{\hat{H}_2}+O(\tau^3). \quad (4)$$

In our arguments, the spatial part in the Hamiltonian is modified whereas the spin part is the same as in the standard formalism. We define the state vector by $\Psi>=|\psi>\otimes|\chi>$, i.e. the tensor product of the spatial state vector $|\psi>$ by the spin state vector $|\chi>$ [23]. Using the fact that

$$\hat{S}|\chi>=\mp g\mu_B B_z|\pm z>,$$

since $\hat{S}_z|\pm z>=\pm\frac{\hbar}{2}$, we can use Eq. (4) to write the nonlocal Schrödinger equation as:

$$\hat{H}_{1,\tau}|\psi>\otimes|\pm z>=\mathrm{E}|\psi>\otimes|\pm z>, \quad (5)$$

where $\mathrm{E}=E\pm g\mu_B B_z$. The Schrödinger equation in the space variables is obtained simply by multiplying Eq. (5) from the left by $<\vec{r}|\otimes<\vec{\omega}|$ where $<\vec{\omega}|\pm z>$ is the spin wave function and plugging in $\hat{H}_{1,\tau}$ which gives after algebra:

$$\frac{1}{2m}\left(\left(\hbar\nabla+q\vec{A}\right)^2-\frac{\hbar^2c^2\tau^2}{2\alpha^2}\Delta^2\right)\psi(\vec{r},t)=\mathrm{E}\psi(\vec{r},t). \quad (6)$$

Using the Landau gauge $\vec{A}=xB_z\vec{y}$, we can write Eq. (6) as:

$$\frac{1}{2m}\left(-\hbar^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+2i\hbar qB_zx\frac{\partial}{\partial y}+q^2B_z^2x^2\right.$$
$$\left.-\frac{\hbar^2c^2\tau^2}{2\alpha^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2\right)\psi(\vec{r},t)=\mathrm{E}\psi(\vec{r},t). \quad (7)$$

Using the possible ansatz is $\psi(\vec{r},t)\equiv\psi=\phi(x)e^{i(k_yy+k_zz)}$ [23] we get:

$$-\frac{\hbar^2c^2\tau^2}{4m\alpha^2}\phi^{(4)}-\left(1-\frac{c^2\tau^2}{2\alpha^2}\left(k_y^2+k_z^2\right)\right)\frac{\hbar^2}{2m}\phi''$$
$$+\frac{q^2B_z^2}{2m}\left(x-\frac{\hbar k_y}{qB_z}\right)^2\phi=\left(\mathrm{E}-\frac{\hbar^2k_z^2}{2m}\right)\phi\equiv E\phi. \quad (8)$$

Surprisingly, this is the equation of a 4th-order oscillator which can be written after neglecting terms on τ^4 as:

$$-\frac{\hbar^2c^2\tau^2}{4m\alpha^2}\phi^{(4)}-\frac{\hbar^2}{2m}\phi''$$
$$+\frac{q^2B_z^2}{2m}\left(1-\frac{c^2\tau^2}{2\alpha^2}\left(k_y^2+k_z^2\right)\right)\left(x-\frac{\hbar k_y}{qB_z}\right)^2\phi$$
$$=\left(\mathrm{E}-\frac{\hbar^2k_z^2}{2m}\left(1-\frac{c^2\tau^2}{2\alpha^2}\left(k_y^2+k_z^2\right)\right)\right)\phi\equiv E\phi, \quad (9)$$

where

$$m\approx m\left(1+\frac{c^2\tau^2}{2\alpha^2}\left(k_y^2+k_z^2\right)\right)+O(\tau^3), \quad (10)$$

being the effective mass. We can compare Eq. (9) with the equation of the 4th-order quantum relativistic oscillator which usually takes the form:

$$-\frac{\hbar^4}{8m^3c^2}\phi^{(4)}-\frac{\hbar^2}{2m}\phi''+\frac{1}{2}m\omega^2(x-x_0)^2\phi+mc^2\phi=E\phi. \quad (11)$$

The energy levels of Eq. (11) may be obtained using the method of Fourier Hermite series [24] and are given by:

$$E_n=mc^2+\hbar\omega\left(n+\frac{1}{2}\right)-\frac{\hbar^2\omega^2}{8mc^2}\left(\frac{3n^2+15n}{2}+\frac{15}{4}\right),n\in \quad (12)$$

Comparing Eqs. (9) and (11) we get:

$$E_n\approx mc^2+\hbar\frac{qB_z}{m}\left(1-\frac{c^2\tau^2}{4\alpha^2}\left(k_y^2+k_z^2\right)\right)\left(n+\frac{1}{2}\right)$$
$$-\frac{\hbar^2q^2B_z^2}{8m^3c^2}\left(\frac{3n^2+15n}{2}+\frac{15}{4}\right), \quad (13)$$

and consequently we find:

$$E_n=mc^2+\left(\frac{\hbar qB_z}{m}\left(n+\frac{1}{2}\right)+\frac{\hbar^2k_z^2}{2m}\right)\left(1-\frac{c^2\tau^2}{4\alpha^2}\left(k_y^2+k_z^2\right)\right)$$
$$-\frac{\hbar^2q^2B_z^2}{8m^3}\left(\frac{3n^2+15n}{2}+\frac{15}{4}\right)\mp g\mu_BB_z+O(\tau^3). \quad (14)$$

This relation differs from the standard result and in terms of m it can be approximate by:

$$E_n=\left(mc^2-\frac{\hbar^2q^2B_z^28m^3}{}\left(\frac{3n^2+15n}{2}+\frac{15}{4}\right)\right)\left(1+\frac{c^2\tau^2}{2\alpha^2}\left(k_y^2+k_z^2\right)\right)$$
$$+\left(\frac{\hbar qB_z}{m}\left(n+\frac{1}{2}\right)+\frac{\hbar^2k_z^2}{2m}\right)\mp g\mu_BB_z+O(\tau^3). \quad (15)$$

Eq. (15) gives rise to another effective nonlocal mass which takes the following form:

$$M=\frac{mc^2\tau^2}{2\alpha^2}\left(k_y^2+k_z^2\right). \quad (16)$$

This mass is independent of the magnetic field and depends on the components of the wave vector in the xy-plane. Numerically, if we consider N-atoms in a solid separated by a lattice spacing L each (for solid Copper $L\approx3.61\times10^{-10}$ m) [25], then usually $k=\frac{2\pi}{\lambda}=\frac{\pi}{L}$ and besides for macroscopic solids at ambient temperature $\tau\approx10^{-11}$ s [26], we obtain for the case of the electron $M\approx3.4\times10^{-20}\alpha^{-2}$ eV/c^2. For $\alpha=10^{-1}$, we find $M\approx3.4\times10^{-18}$ eV/c^2 which is close to the mass of a photon [27]. The following statement then holds:

Statement 1: The nonlocal-in-time (kinetic energy) quantization of a charged particle in a magnetic field with spin is governed