Journal of Magnetism and Magnetic Materials 452 (2018) 30–34

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Magnon condensation and spin superfluidity

Yury M. Bunkov [a], Vladimir L. Safonov [b,c,*]

[a] Kazan Federal University, Kremlevskaya 18, 420008 Kazan, Russia
[b] Mag and Bio Dynamics, Inc., Granbury, TX 76049, USA
[c] Physical Science Department, Tarrant County College, South Campus, Fort Worth, TX 76119, USA

ARTICLE INFO

Article history:
Received 24 August 2017
Received in revised form 12 November 2017
Accepted 7 December 2017
Available online 8 December 2017

Keywords:
Bose-Einstein condensation
Magnons
YIG
Hematite

ABSTRACT

We consider the Bose-Einstein condensation (BEC) of quasi-equilibrium magnons which leads to spin superfluidity, the coherent quantum transfer of magnetization in magnetic material. The critical conditions for excited magnon density in ferro- and antiferromagnets, bulk and thin films, are estimated and discussed. It was demonstrated that only the highly populated region of the spectrum is responsible for the emergence of any BEC. This finding substantially simplifies the BEC theoretical analysis and is surely to be used for simulations. It is shown that the conditions of magnon BEC in the perpendicular magnetized YIG thin film is fulfillied at small angle, when signals are treated as excited spin waves. We also predict that the magnon BEC should occur in the antiferromagnetic hematite at room temperature at much lower excited magnon density compared to that of ferromagnetic YIG. Bogoliubov's theory of Bose-Einstein condensate is generalized to the case of multi-particle interactions. The six-magnon repulsive interaction may be responsible for the BEC stability in ferro- and antiferromagnets where the four-magnon interaction is attractive.

1. Introduction

Spin deviations from the magnetic order in a magnetic material (ferromagnet, antiferromagnet or ferrites) are manifested by spin waves and their quanta, magnons. Magnons are quasiparticles which represent a very useful quantum theoretical tool to describe various dynamic and thermodynamic processes in magnets in terms of magnon gas. Since magnons have magnetic moments, the external alternating magnetic field can excite extra magnons and increase the disorder in the magnetic system. However, in certain conditions, the increase of magnon density leads to a new state, so-called, magnon condensate, in which a macroscopic number of magnons forms a coherent quantum state (see, e.g., [1,2]). This macroscopic state can significantly change the properties of magnon gas, its dynamics and transport. An example is the phenomenon of quasi-equilibrium Bose-Einstein condensation (BEC) of excited magnons on the bottom of their spectrum as a single-particle long-range coherent state of quantum liquid. This state generate an uniform long-lived precession of spins formed by quantum specificity of the magnon gas when the magnon density exceeds certain critical value. The spatial gradients of this state exhibit a spin superfluidity, the non-potential transport of deflected magnetization. The spin superfluidity is an extremely interesting phenomenon for both fundamental and applied studies. It should be emphasized that the main paradigm of magnetic dynamics, the Landau-Lifshitz-Gilbert equation, does not contain complete information about the Bose-Einstein condensate of magnons. BEC is the principal result of quantum statistics and for magnons it can exist at room and even higher temperatures.

For the first time the existence of quasi-equilibrium Bose condensate was demonstrated in the experiment with nuclear magnons in the superfluid antiferromagnetic liquid crystal ^{3}He-B in 1984 [3]. The theoretical explanation of this phenomenon [4] was developed on the basis of global Ginzburg-Landau energy potential. A similar approach was later developed to explain the atomic BEC [5]. In the experiments with an antiferromagnetic ^{3}He-B, the following phenomena were observed: a) transport of magnetization by spin supercurrent between two cells with magnon BEC; b) phase-slip processes at the critical current; c) spin current Josephson effect; d) spin current vortex formation; d) Goldstone modes of magnon BEC oscillations. Comprehensive reviews of these studies can be found in Refs.[6–8]. Currently magnon BEC found in different magnetic systems: i) in antiferromagnetic superfluid ^{3}He-A [9,10]; ii) in in-plane magnetized yttrium iron garnet $Y_3Fe_5O_{12}$ (YIG) film (with two minima in the magnon spectrum) [11,12] and in normally magnetized YIG film [13]; iii) in antiferromagnets $MnCO_3$ and $CsMnF_3$ with Suhl-Nakamura indirect nuclear spin-spin interaction [14–16]. An explanation of analogy between the atomic and magnon BEC is given in Ref. [17].

* Corresponding author.
E-mail address: vlsafonov@magbiodyn.com (V.L. Safonov).

https://doi.org/10.1016/j.jmmm.2017.12.029
0304-8853/

A microscopic theory of quasi-equilibrium magnon BEC was developed in Refs. [18–21] ("KS theory"). It was predicted that the external strong pumping of magnons leads to a rapid growth of magnon density and saturation. This state can be considered in terms of weakly non-ideal gas of "dressed" magnons in a thermodynamic quasi-equilibrium with an effective chemical potential μ and effective temperature T. The dressed magnon energy is defined by $\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}}^{(0)} + \delta\varepsilon_{\mathbf{k}}$, where $\varepsilon_{\mathbf{k}}^{(0)} = \hbar\omega_{\mathbf{k}}$ is the energy spectrum of bare magnons and $\delta\varepsilon_{\mathbf{k}}$ is the energy shift due to magnon gas nonlinearities. Magnon-magnon scattering processes retain the total number of dressed magnons in the system and hold their distribution function of the form

$$n_{\mathbf{k}} = \left(\exp\frac{\varepsilon_{\mathbf{k}} - \mu}{k_B T} - 1\right)^{-1}. \quad (1)$$

The instability at $\mu = \min \varepsilon_{\mathbf{k}}$ in the quasi-equilibrium magnetic system is an analog of BEC phenomenon for the bottom dressed magnons. The distribution (1) seems to underlie the phenomenon of spin superfluidity, since it nullifies the integral of four-magnon collisions

$$I^{(4)}\{n_{\mathbf{k}}\} \propto \int d^3k_1 d^3k_2 d^3k_3 |\Phi_4(\mathbf{k}, \mathbf{k}_1; \mathbf{k}_2, \mathbf{k}_3)|^2 \times [(n_{\mathbf{k}} + 1)(n_{\mathbf{k}_1} + 1)n_{\mathbf{k}_3} n_{\mathbf{k}_4} - n_{\mathbf{k}} n_{\mathbf{k}_1}(n_{\mathbf{k}_2} + 1)(n_{\mathbf{k}_3} + 1)] \times \delta(\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}_1} - \varepsilon_{\mathbf{k}_2} - \varepsilon_{\mathbf{k}_3})\Delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \quad (2)$$

and thus this energy loss channel vanishes.

KS theory qualitatively explained the parallel pumping experiments ([22,23] (YIG at room temperature) and [24] (nuclear magnons in $CsMnF_3$), where the accumulation of magnons at the bottom of the spin wave spectrum was observed. One and a half decade later, purposeful experiment [11] directly demonstrated BEC of quasi equilibrium magnons in the thin film of YIG. Subsequent experimental studies have shown qualitative correspondence with the predicted distribution of excited magnons [25] and agreement with the BEC under noisy pumping [26].

In this paper we analyze critical conditions of quasi-equilibrium magnon BEC in ferro- and antiferromagnets, bulk and thin films, and evaluate the possibilities of their experimental achievements.

2. BEC of bose particles

Let us first briefly discuss BEC of real bose particles. Their distribution is defined by Eq. (1), where $\varepsilon_k = (\hbar k)^2/2m$ is the kinetic energy of particle with the wave vector k and mass m. The total number of bosons in the system is

$$N(\mu, T) = N = V_s \int n_{\mathbf{k}} \frac{d^3k}{(2\pi)^3}, \quad (3)$$

where V_s is the volume of the system. For the critical condition $\mu = \min \varepsilon_k$, from (3) follows well-known formula for the BEC critical temperature versus the density of bosons:

$$T_{BEC} = \kappa_0 \frac{\hbar^2}{k_B m}\left(\frac{N}{V_s}\right)^{2/3}, \quad \kappa_0 = \frac{2\pi}{\left[\zeta\left(\frac{3}{2}\right)\right]^{2/3}} \simeq 3.31. \quad (4)$$

It is interesting to note that the BEC is formed mainly by bosons with high populations when Eq. (1) can be written as

$$n_k \simeq \frac{k_B T}{\varepsilon_k - \mu}. \quad (5)$$

Let us prove it by direct calculation. Substituting the high temperature population (5) into Eq. (3) and cutting the upper integral limit by the thermal energy $\varepsilon_T \simeq k_B T$, one obtains:

$$T_{BEC} \simeq \tilde{\kappa}_0 \frac{\hbar^2}{k_B m}\left(\frac{N}{V_s}\right)^{2/3}, \quad \tilde{\kappa}_0 = \frac{\pi^{4/3}}{2^{1/3}} \simeq 3.65. \quad (6)$$

We see that the only difference between Eqs. (4 and (6) is a slightly different ($\sim 10\%$) numerical factor.

The fact that the high population Eq. (5) is dominant does not mean that the BEC phenomenon is a classical one. The criterion of classical Maxwell-Boltzmann statistics $\exp(\mu/k_B T) \ll 1$ in this case can be written as (see, e.g., [27]):

$$\exp(\mu/k_B T) = \left[\frac{V_s}{N}\int \exp\left(-\frac{\varepsilon_{\mathbf{k}}}{k_B T}\right)\frac{d^3k}{(2\pi)^3}\right]^{-1} \ll 1, \quad (7)$$

or

$$\frac{N}{V_s}\lambda_T^3 = \frac{N}{V_s}\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{3/2} \ll 1, \quad (8)$$

where λ_T is the thermal de Broglie wavelength. Substituting BEC temperature Eqs. (6) into (8), we obtain the opposite relation: $2.26 > 1$, which obviously corresponds to a degenerate bose gas.

3. BEC of magnons

Now let us consider a Bose-Einstein condensation of so-called, "dressed" magnons as an instability in the externally pumped quasi-equilibrium magnon gas. The total number of magnons $N(\mu, T)$ is equal to the number of thermal magnons $N(0, T)$ at a given temperature T and the number of magnons N_p created by external pumping. So far as the energy shift of dressed magnons is usually much less than the energy of bare magnons $\delta\varepsilon_k \ll \min \varepsilon_k^{(0)}$, for simplicity we can approximate $\varepsilon_k \simeq \varepsilon_k^{(0)}$.

3.1. BEC in a ferromagnet

Consider a ferromagnet with the quadratic spectrum (we neglect details of the dipole-dipole interactions):

$$\varepsilon_k = \varepsilon_0 + \varepsilon_{ex}(ak)^2. \quad (9)$$

Here ε_{ex} is the exchange interaction constant and a is the elementary cell linear size. The quasi-equilibrium BEC will be mainly determined by pumping if the number of pumped magnons is much greater than the thermal magnon number $N_p \gg N(0, T)$. In this case we obtain an analog of Eq. (4):

$$T_{BEC} = \kappa_0 \frac{2\varepsilon_{ex}}{k_B}\left(a^3 \frac{N_p}{V_s}\right)^{2/3}, \quad (10)$$

or,

$$T_{BEC} \simeq \tilde{\kappa}_0 \frac{2\varepsilon_{ex}}{k_B}\left(a^3 \frac{N_p}{V_s}\right)^{2/3} \quad (11)$$

in the high-population approximation.

The above formula, however, does not work for a BEC estimate if $N_p \lesssim N(0, T)$. Using a high-population approximation, we write

$$N_p = N(\mu, T) - N(0, T) \simeq V_s \int_{\varepsilon_0}^{\varepsilon_T}\left(\frac{k_B T}{\varepsilon_k - \mu} - \frac{k_B T}{\varepsilon_k}\right)\frac{k^2 dk}{2\pi^2}, \quad (12)$$

and obtain at $\mu = \varepsilon_0$

$$\frac{N_p}{V_s} \simeq \frac{k_B T_{BEC}}{4\pi a^3}\frac{\varepsilon_0^{1/2}}{\varepsilon_{ex}^{3/2}},$$

$$T_{BEC} \simeq 4\pi \frac{\varepsilon_{ex}}{k_B}\left(\frac{\varepsilon_{ex}}{\varepsilon_0}\right)^{1/2}\left(a^3 \frac{N_p}{V_s}\right). \quad (13)$$