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Haar wavelet collocation method for the numerical solution of singular initial value problems

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Abstract In this paper, numerical solutions of singular initial value problems are obtained by the Haar wavelet collocation method (HWCM). The HWCM is a numerical method for solving integral equations, ordinary and partial differential equations. To show the efficiency of the HWCM, some examples are presented. This method provides a fast convergent series of easily computable components. The errors of HWCM are also computed. Through this analysis, the solution is found on the coarse grid points and then converging toward higher accuracy by increasing the level of the Haar wavelet. Comparisons with exact and existing numerical methods (adomian decomposition method (ADM) & variational iteration method (VIM)) solutions show that the HWCM is a powerful numerical method for the solution of the linear and non-linear singular initial value problems. The Haar wavelet adaptive grid method (HWAGM) based solutions show the excellent performance for the proposed problems.

1. Introduction

In recent years, the studies of singular initial value problems in the second order ordinary differential equations (ODEs) have attracted the attention of many mathematicians and physicists. Many methods including numerical and perturbation methods have been used to solve such type of problems. The approximate solutions for these problems were presented by many researchers for example Wazwaz [1–3] using the ADM and Yildirim and Ozis [4] using the VIM.

In numerical analysis, classical discretization methods, such as finite differences, finite elements and spectral elements are powerful tools for solving differential equations. However, singularities and step changes often emerge in many phenomena, such as stress concentration, elastoplasticity, shock wave and crack. Since small-scale features only exist in a small percentage of the solution domain, if one chooses a uniform numerical grid fine enough to resolve the small-scale characteristics, then the solution to the equations will be over-resolved in the majority of the domain. One would like ideally, to have a dense grid where small-scale structure exists and a sparse grid where the solution is only composed of large-scale features

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[5–7]. It demands for the usage of non-uniform grids and adaptive grids or moving elements to dynamically adapt to the changes in the solution [8]. That is where wavelets play a role.

Wavelet is called "numerical microscope" in signal and image processing. It has been 31 years since Morlet proposed the concept of wavelet analysis to automatically reach the best trade-off between time and frequency resolution [9]. Later, this proposition was considered as a generalization of ideas promoted by Haar (1910), Gabor (1946) [10]. Wavelet was in the air in the numerical analysis community in the early 1990s [11]. Generally, wavelet is used to describe a function that features compact support; i.e. it is nonzero only on a finite interval. The representation of a set of time-dependent data on a wavelet basis leads to a unique structure of information that is localized simultaneously in the frequency and time domains. This does not occur in a Fourier representation, where specific frequencies cannot be associated with a particular time interval, since the basis functions have constant resolution on the entire domain. A wavelet basis representation originates a set of wavelet coefficients structured over different levels of resolution. Each coefficient is associated with a resolution level and a point in the time domain. The coefficients involved in the lowest-resolution level describe the low-frequency features of the data spanning over broad time intervals. At the highest level, the coefficients are associated with highly localized high-frequency features [12,13]. These desirable advantages draw sight of researchers to apply wavelets in the resolution of differential equations [14–16].

One of the popular families of wavelets is Haar wavelet. Due to its simplicity, Haar wavelet has become an effective tool for solving differential equations. The previous work in system analysis via Haar wavelet was led by Chen and Hsiao [17], who first derived a Haar operational matrix for the integrals of the Haar function vector and put the applications for the Haar analysis into the dynamic systems. Lepik [18–20] applied the Haar wavelet collocation method for the solution of differential and integral equations. Bujurke et al. [21–23] presented the Haar wavelet method to establish the solution of nonlinear oscillator equations, Stiff systems, regular Sturm–Liouville problems, etc. Chang and Piau [24], designed the numerical solution of ordinary differential equations using Haar wavelet matrices. Islam et al. [25] obtained the numerical solution of second-order boundary-value problems using the Haar wavelet collocation method for the different boundary conditions.

The purpose of this paper is to introduce the HWCM as an alternative to existing methods for solving singular initial value problems. With this method, the given differential equation and its related initial conditions are transformed into a recurrence equation that finally leads to the solution of a system of algebraic equations as coefficients of a Haar series solution. This method is useful to obtain the approximate solutions of linear and nonlinear singular initial value problems, no need to linearization or discretization and large computational work. It has been used to solve effectively, easily and accurately a large class of linear and nonlinear problems with approximations.

The present work is organized as follows. In Section 2, Haar wavelet and operational matrix of integration are given. Method of solution of HWCM is presented in Section 3. In Section 4 numerical results and error analysis of the test problems are obtained. Finally conclusion of the proposed work is discussed in Section 5.

2. Haar wavelet and operational matrix of integration

The scaling function $h_1(t)$ for the family of the Haar wavelet is defined as

$$h_1(t) = \begin{cases} 1 & \text{for } t \in [0,1) \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

The Haar wavelet family for $t \in [0,1)$ is defined as

$$h_i(t) = \begin{cases} 1 & \text{for } t \in \left[\frac{k}{m}, \frac{k+0.5}{m}\right) \\ -1 & \text{for } t \in \left[\frac{k+0.5}{m}, \frac{k+1}{m}\right) \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

where $m = 2^l$, $l = 0, 1, \ldots, J$, J is the level of resolution; and $k = 0, 1, \ldots, m-1$ is the translation parameter. Maximum level of resolution is J. The index i in Eq. (2.2) is calculated using $i = m + k + 1$. In case of minimal values $m = 1$, $k = 0$ then $i = 2$. The maximal value of i is $N = 2^{J+1}$.

Let us define the collocation points $t_j = \frac{j-0.5}{N}$, $j = 1, 2, \ldots, N$, Haar coefficient matrix $H(i,j) = h_i(t_j)$ which has the dimension $N \times N$. For instance, $J = 3 \Rightarrow N = 16$, then we have

$$\mathrm{H}(16,16) = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{pmatrix}$$

We establish an operational matrix for integration via Haar wavelet. The operational matrix of integration is obtained by integrating (2.2) is as,

$$Ph_i = \int_0^t h_i(t)dt \tag{2.3}$$

and

$$Qh_i = \int_0^t Ph_i(t)dt \tag{2.4}$$

These integrals can be evaluated by using Eq. (2.2) and they are given by

$$Ph_i(t) = \begin{cases} t - \frac{k}{m} & \text{for } t \in \left[\frac{k}{m}, \frac{k+0.5}{m}\right) \\ \frac{k+1}{m} - t & \text{for } t \in \left[\frac{k+0.5}{m}, \frac{k+1}{m}\right) \\ 0 & \text{otherwise} \end{cases} \tag{2.5}$$