Ain Shams Engineering Journal (2016) 7, 671–682

Ain Shams University

Ain Shams Engineering Journal

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ENGINEERING PHYSICS AND MATHEMATICS

Mixed convection flow due to a vertical plate in the presence of heat source and chemical reaction

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Received 22 December 2014; revised 31 March 2015; accepted 13 May 2015
Available online 3 July 2015

KEYWORDS

MHD flow;
Heat and mass transfer;
Internal heat generation;
Convective boundary condition;
HAM solution

Abstract In this paper, the hydromagnetic heat and mass transfer by mixed convection flow due to a vertical flat plate is considered for analysis. The governing equations are solved both analytically and numerically. The analytical solutions are obtained using the Homotopy Analysis Method (HAM) while the numerical solutions are computed using Keller–Box method (K–B). Convergence of the Homotopy solutions for the governing non-dimensional equations are derived. A detailed error analysis is done to compute the average squared residual errors for flow, temperature and concentration. The optimal values of the convergence control parameter are computed for velocity and temperature. This study includes the effects of various parameters such as magnetic parameter, Grashof number, chemical reaction parameter, heat source parameter and Biot number on skin friction, heat and mass transfer rates as well on velocity, temperature and concentration profiles. Comparison of the HAM and K–B methods shows a very good agreement.

1. Introduction

The study of boundary layer flow with heat and mass transfer over a vertical plate, in the presence of magnetic field and chemical reaction has important applications in various chemical and engineering processes such as petroleum and chemical industry, thermal insulation, cooling of nuclear reactors [1]. The convective heat transfer studies are very useful in understanding processes involving high temperature and concentration, for example, gas turbines and thermal energy storage. Other promising applications are in the field of metallurgy such as MHD stirring of molten metal and magnetic-levitation casting. Makinde [2,3] studied the heat and mass transfer in the presence of magnetic field due to a moving vertical plate with a convective surface boundary condition effects numerically using higher order Runge Kutta Scheme. The flow due to a semi-infinite vertical plate with convective surface boundary condition and internal heat generation in the presence of viscous dissipation, thermal radiation and chemical reaction was studied numerically by [4]. Aziz [5] has investigated the different variations of laminar thermal boundary layer and thermal slip flow due to a flat plate in a uniform stream of fluid with a convective surface and constant heat flux boundary conditions. Most of these studies pertain to numerical methods using Runge Kutta scheme.

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Peer review under responsibility of Ain Shams University.

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http://dx.doi.org/10.1016/j.asej.2015.05.008
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Recently, Anuar et al. [6] studied the effect of radiation on the thermal boundary layer flow due to a moving plate. Olanrewaju et al. [7] presented the effects of thermal diffusion, magnetic field and viscous dissipation on unsteady mixed convection flow past a porous plate with chemical reaction. Thermal radiation with buoyancy force on the MHD fluid flow on the permeable vertical stretching sheet was considered by Rashidi et al. [8].

The analytical and numerical solutions in a steady two-dimensional flow of viscous fluid with chemical reaction and heat generation were considered by Shatey et al. [9]. Shehzad et al. [10] included the Casson fluid flow with chemical reaction and suction effects for heat and mass transfer on MHD flow.

The hydromagnetic boundary layer flow with heat and mass transfer due to a vertical plate in the presence of chemical reaction, magnetic field and a convective heat exchange at the surface was examined numerically by Gangadhar [11] and Gangadhar et al. [12]. Both the works deal with the numerical solution using Runge–Kutta fourth order with shooting technique. In [11], the author has not considered the source parameter (Sr) and the concentration difference parameter (Nc), where as in [12], he has included the source parameter (Sr) but not considered the concentration difference parameter (Nc).

The focus of this paper is to obtain an approximate analytical solution for the hydromagnetic flow due a vertical plate in the presence of a heat source and chemical reaction which has not been taken up for analysis in the literature. The flow is subjected to a convective surface boundary condition in the presence of local and modified thermal Grashof numbers and the study includes the combined effects of source parameter and the concentration difference parameter. The non linear equations governing the flow were solved analytically using Homotopy Analysis Method (HAM). The study may resemble the work of Gangadhar [11] and Gangadhar et al. [12], but both these studies are only numerical approach using Runge Kutta fourth order with shooting technique whereas the present analysis is an approximate analytical solution procedure. The governing equations and the boundary conditions imposed on concentration are different from that of Gangadhar [11] and Gangadhar et al. [12] and the present study takes into account both heat source parameter (Sr) and Concentration difference parameter (Nc) to study the effect of magnetic and internal heat generation on the fluid. Convergence of the Homotopy solutions is also derived.

2. Problem formulation and governing equation

Here, we study the mixed convection flow of a laminar, incompressible hydromagnetic flow due to a vertical plate in the presence of a heat source and chemical reaction. The fluid is subjected to a uniform magnetic field of strength B_0. It is assumed that the magnetic Reynolds number of the flow is assumed to be small enough so that the induced distortion due to applied magnetic field can be neglected. On the assumption of a small magnetic Reynolds number, heat energy dissipated through viscous dissipation and the heat released due to ohmic heating are negligible compared to the energy due to heat source since our surface is heated by convection. Hence the combined effects of viscous dissipation and Ohmic heating due to magnetics are neglected in the energy equation.

Under these assumptions, the governing equations for the steady two dimensional boundary layer flow due to a vertical plate are given by [2]

$$u_x + v_y = 0 \tag{1}$$

$$uu_x + vu_y = \nu u_{yy} - \frac{\sigma B_0^2}{\rho}(u - U_\infty) + g_1\beta_1(T - T_\infty) + g_1\beta_2(C - C_\infty) \tag{2}$$

$$uT_x + vT_y = \alpha T_{yy} + \frac{Q_0}{\rho C_p}(T - T_\infty) \tag{3}$$

$$uC_x + vC_y = DC_{yy} - Kr_1 C \tag{4}$$

The boundary conditions are

$$u = v = 0, \quad -kT_y = h_c\left(T_f - T\right), \quad C_w = Ax^\beta + C_\infty \quad \text{at } y = 0,$$

$$u \to U_\infty, T \to T_\infty \text{ and } C \to C_\infty \text{ as } y \to \infty. \tag{5}$$

The coordinate system for the two-dimensional flow is given in Fig. 1. The x-axis is considered along the direction of the plate and the y-axis is normal to it. A uniform magnetic field of strength B_0 is applied in the downward y-axis. The left surface of the plate is being heated by convection from a hot fluid and T_f represents the left side of the surface plate of temperature that gives a heat transfer coefficient h_c.

Here u and v denote the fluid velocity along the x- and y- directions, respectively; T is the temperature; C is the concentration; k is the thermal conductivity; ρ is the fluid density; ν is the kinematic viscosity; α, D are the thermal and mass diffusivity respectively; β_1 and β_2 are the thermal and solutal expansion coefficients; β is the power index of the concentration; σ is the fluid electrical conductivity; Q_0 is the heat source; Kr_1 is the chemical reaction rate on the species concentration and g_1 is the gravitational acceleration; A is the constant and C_w is the species concentration at the plate; C_p is the specific heat at a constant pressure and U_∞, T_∞ and C_∞ are the free stream velocity, temperature and concentration respectively.

The stream function ψ, satisfies the Eq. (1) automatically with

$$u = \frac{\partial\psi}{\partial y}; \quad v = -\frac{\partial\psi}{\partial x} \tag{6}$$

It may be remarked that the partial differential Eqs. (1)–(6) are obtained by defining an independent variable η and the stream function ψ as follows

$$\eta = y\left(\frac{U_\infty}{\nu x}\right)^{1/2}; \quad \psi = (\nu x U_\infty)^{1/2} f(\eta) \tag{7}$$

The dimensionless temperature and concentration are denoted as

$$T - T_\infty = (T_f - T_\infty)g(\eta); \quad C - C_\infty = (C_w - C_\infty)G(\eta) \tag{8}$$

Substituting Eqs. (6)–(8) into Eqs. (1)–(5), we get following coupled nonlinear ODEs;

$$f'''(\eta) + \frac{1}{2}f(\eta)f''(\eta) + Ha(1 - f'(\eta)) + Gr\ g\ (\eta) + Gc\ G(\eta) = 0 \tag{9}$$

$$g''(\eta) + \frac{1}{2}Pr\ f\ (\eta)g'(\eta) + PrSrg(\eta) = 0 \tag{10}$$