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A novel computing three-dimensional differential transform method for solving fuzzy partial differential equations

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Abstract In this paper, we introduce three-dimensional fuzzy differential transform method and we utilize it to solve fuzzy partial differential equations. This technique is a successful method because of reducing such problems to solve a system of algebraic equations; so, the problem can be solved directly. A considerable advantage of this method is to obtain the analytical solutions if the equation has an exact solution that is a polynomial function. Numerical examples are included to demonstrate the validity and applicability of the method.
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1. Introduction

The idea of fuzzy set was first introduced by Latfi Zadah in 1960 as means of handling uncertainty that is due to impression or vagueness rather than to randomness. Definition of fuzzy sets which was proposed in [1], led to the definition of the fuzzy number and its implementation in fuzzy control [2] and approximate reasoning problems [3,4]. The basic arithmetic structure for fuzzy numbers was later developed in papers [5–9].

Recently due to industrial interest in fuzzy control, the applications of the theory of fuzzy differential and integral equation have been increased. Fuzzy partial differential equations (FPDEs) play a major role in modeling uncertainty of dynamical systems. Hence, studies of PDEs have become one of the main topics of modern mathematical analysis and have attracted much attention. However, the exact solutions to the PDEs cannot be easily obtained except for very simple or special cases. In recent years, many methods have been developed for solving some kinds of PDEs. Some mathematicians have studied solution of FPDE by numerical methods [10–14].

Differential transform method is different from the traditional high order Taylor series method, which requires symbolic computation of necessary derivatives of the data function and is computationally expensive for higher order. The concept of differential transform (one-dimension) was proposed and applied to solve linear and nonlinear initial value problems in electric circuit analysis by Zhou [15].

The topic of fuzzy differential transform method has been rapidly grown in recent years. The aim of this paper is using

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three-dimensional fuzzy differential transform method (FDTM) to solve FPDEs. The FDTM evaluates the approximating solution by finite Taylor series.

This paper is organized as follows: first, we introduce some fuzzy concepts and their properties. Three-dimensional FDTMs and their properties are introduced in Section 3. In Section 4, we apply three-dimensional fuzzy differential transform method to solve fuzzy partial differential equations by illustrating some numerical examples to show the accuracy and advantages of this method. Finally Section 5 concludes the paper.

2. Preliminaries

2.1. Fundamental operations

Differential transform method is different from the traditional high-order Taylor series method, which requires symbolic computation of necessary derivatives of the data function and is computationally expensive for higher order. The basic definitions of the three-dimensional transform are defined as follows:

$$W(i,j,k)=\frac{1}{i!j!k}\left[\frac{\partial^{i+j+k}w(x,y,z)}{\partial x^i\partial y^j\partial z^k}\right]_{(0,0,0)}, \tag{1}$$

where $w(x,y,z)$ is the original function and $W(i,j,k)$ is the transformed function. The transformation is called T-function and the lower case and upper case letters represent the original and transformed functions respectively. The differential inverse transform of $W(i,j,k)$ is defined as

$$w(x,y,z)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}W(i,j,k)x^iy^jz^k, \tag{2}$$

and from Eqs. (1) and (2) can be concluded

$$w(x,y,z)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{i!j!k!}\left[\frac{\partial^{i+j+k}w(x,y,z)}{\partial x^i\partial y^j\partial z^k}\right]_{(0,0,0)}. \tag{3}$$

Table 1 contains some fundamental operations of three-dimensional differential transform method.

2.2. Fuzzy concepts

In this section we will recall some basics definitions and theorems needed throughout the paper such as fuzzy number, fuzzy-valued function and the derivative of the fuzzy-valued functions.

Definition 1 [17]. A fuzzy number $\tilde{w}$ is convex normalized fuzzy set $\tilde{w}$ of the real line R such that

$$\{\mu_{\tilde{w}}:R\to[0,1],\ \forall x\in R\},$$

where $\mu_{\tilde{w}}$ is called the membership function of the fuzzy set and it is piecewise continuous.

Definition 2 ([18,19]). A fuzzy number is a function $\tilde{w}:R\to[0,1]$ which satisfies the following properties

(i) $\tilde{w}$ is normal, i.e. $\exists\, x_0\in R$ with $\tilde{w}(x_0)=1$,
(ii) $\tilde{w}$ is a convex fuzzy set, i.e. $\tilde{w}(rx+(1-r)y)\geqslant\min\{\tilde{w}(x),\tilde{w}(y)\}$, for any $x,y\in R,\ r\in[0,1]$,
(iii) $\tilde{w}$ is upper semi-continuous,
(iv) $[\tilde{w}]_0=\overline{\{x\in R:\ \tilde{w}(x)>0\}}$ is compact, here $\overline{A}$ denotes the closure of A.

Any real number $a\in R$ can be interpreted as a fuzzy number $\tilde{a}=\chi_{\{a\}}$ and therefore $R\subset E^1$. For any $0\leqslant r\leqslant 1$ we denote the r-level set $[\tilde{w}]_r=\{x\in R:\ \tilde{w}(x)\geqslant r\}$, that is a closed interval [18] and $[\tilde{w}]_r=[w_r^-,w_r^+],\ \forall r\in[0,1]$.

For $\tilde{w}_r,\ \tilde{v}_r\in E^1,\ k\in R$, the addition and the scalar multiplication are defined by $[\tilde{w}+\tilde{v}]_r=[w_r^-+v_r^-,w_r^++v_r^+]$, $\forall r\in[0,1]$

$$k\odot[\tilde{w}]_r=\begin{cases}[kw_r^-,kw_r^+], & k\geqslant 0\\ [kw_r^+,kw_r^-], & k<0\end{cases}.$$

The collection of all the fuzzy numbers with addition and multiplication defined by above equations is denoted by E^1. We will next define the fuzzy function notation and a metric D on E^1 [20].

As a distance between fuzzy numbers we use the Hausdorff metric [18] defined by

$$D(\tilde{w}_r,\tilde{v}_r)=\sup_{0\leqslant r\leqslant 1}\{\max\left(|w_r^--v_r^-|,|w_r^+-v_r^+|\right)\},\qquad \tilde{w}_r,\tilde{v}_r\in E^1.$$

In the following definition, we will introduce the H-derived (differentiability in the sense of Hukuhara) and differential transform method [21].

Definition 3 [22]. Let $\tilde{w}_r,\tilde{v}_r\in E^1$. If there exists $\tilde{u}_r\in E$ such that $\tilde{w}_r=\tilde{v}_r+\tilde{u}_r$, then $\tilde{u}_r$ is called the H-difference of $\tilde{w}_r$ and $\tilde{v}_r$, that is denoted by $\tilde{w}_r\ominus\tilde{v}_r$.

Table 1 The fundamental operations of three-dimensional differential transform method [16].

Original function	Transformed function
$u(x,y,z)=w(x,y,z)\pm v(x,y,z)$	$U(i,j,k)=W(i,j,k)\pm V(i,j,k)$
$u(x,y,z)=cw(x,y,z)$	$U(i,j,k)=cW(i,j,k)$
$u(x,y,z)=\frac{\partial}{\partial x}w(x,y,z)$	$U(i,j,k)=(i+1)W(i+1,j,k)$
$u(x,y,z)=\frac{\partial}{\partial y}w(x,y,z)$	$U(i,j,k)=(j+1)W(i,j+1,k)$
$u(x,y,z)=\frac{\partial^{p+q+t}}{\partial x^p\partial y^q\partial z^t}w(x,y,z)$	$U(i,j,k)=\frac{(i+p)!(j+q)!(k+t)!}{i!j!k!}W(i+p,j+q,k+t)$
$u(x,y,z)=x^iy^jz^k$	$U(\aleph,\Re,\Im)=\delta(\aleph-i,\Re-j,\Im-k)=\begin{cases}1 & \aleph=i,\Re=j,\Im=k\\ 0 & otherwise\end{cases}$

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