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Analysis on phase properties of a transmission line model with non-linear elements

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Abstract The theme of this paper is to analyze phase response of a conventional transmission line (Tx line) unit cell with the presence of a non-linear element. In this analysis, a non-linear inductor and non-linear capacitor are used in two Tx line models, firstly, forward wave supporting structure (low pass in nature) and secondly composite right hand left hand meta-material (CRLH-MTM) structure (bandpass in nature) which supports both forward and backward waves. For the non-linearity, a quadratic constitutive relation is considered between charge and voltage for a capacitor. For an inductor it is between the flux and the current. This analysis is to investigate the possibility of producing the negative group delay in the unit cell of Tx model in the presence of a non-linear element. This implies the opposite phase velocity and group velocities in the dispersion relationship for the Tx line model.

1. Introduction

Linear transmission line theory has been used very extensively in microwave engineering field in designing filters and phase shifters and in almost all passive and active components. Realization of composite right hand left hand meta-materials (CRLH-MTMs) using microstrip is a very active research area in electromagnetics and many devices have been developed so far [1–8]. The main reason of research for these novel circuits is to come up with negative refractive index material or to come up with the conclusion that the phase velocity and group velocity are in opposite direction for a forward travelling wave which is a different phenomenon from conventional positive refractive index material. But the main problem with these circuits when they are realized in the microstrip is that, the gain appears to be very small [9] when the transition is happening from same sign of phase velocity and group velocity to the opposite sign. In this paper it is shown that with the non-linearity in the unit cell, it is possible to avoid this difficulty thereby opening an opportunity for the non-linear transmission lines in a better perspective.

Almost all devices exhibit non-linearity depending on the conditions of operations. But non-linear circuits are very common in electronics field such as clippers, clampers and harmonic generators. In many cases, if there are any non-linear devices present in the circuit or system, they are linearized within its operating region designated as bias point or quiescent point using the Taylor's series expansion [10]. In some applications the circuits are supposed to be analyzed without

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any linearization to see the actual effects of non-linearity in the entire system. There are so many different reasons for the non-linearity for any given component. Capacitor with fringing field can be modelled as a non-linear capacitor, inductor on an iron core is a non-linear device and resistor in an environment with changing temperature is considered as non-linear resistance. Sometimes these non-linearities maybe very complicated functions of controlling parameters. Even though there are infinitely many possibilities for the non-linearity of a device, it is appropriate to fundamentally study the effect of a specific non-linearity for observing the systems behaviour to some extent.

This paper starts with modelling an inductor on iron core as a non-linear element with quadratic constitutive relation between its controlling parameters (magnetic flux and current) [11]. Two transmission line models are analyzed in time domain with the Gaussian pulse as its excitation. Then with the help of Fourier Transform theory [12], its phase response in frequency domain is obtained. First circuit is with the low pass structure and the second one is the composite right hand left hand meta-material. Along with this a five section LC transmission line model is also analyzed to see the response of this in circuit domain to see the modified low pass behaviour as a function on non-linearity.

2. Problem formulation

The aim of this paper is to analyze the effects of a non-linearity on the phase delay and group delay for forward wave supporting structure and forward–backward supporting structure. Fig. 1 represents the conventional transmission line model. For a forward wave supporting structure, the series branch is an inductor and shunt branch is a capacitor forming the well known LC ladder network. For forward and backward wave supporting structure, the series branch is a combination of capacitor and inductor connected in series while the shunt branch is the parallel combination of capacitor and inductor. Linear circuit analysis is very easy compared with the non-linear circuit analysis. A linear circuit or system's behaviour can be expressed completely in terms of unit impulse response, if it is assumed to be time invariant. This is not the case with non-linear systems. In analyzing the non-linear circuits or systems, there are problems such as frequency conversion, harmonics generations and hence it is not possible to come up the output calculation for a given circuit from its impulse response as it is not an LTI system. As far as finding the frequency response for a non-linear device is concerned, depending on the spectrum of input signal, it is possible to find the ratio of output to input, if both (input and output) the power

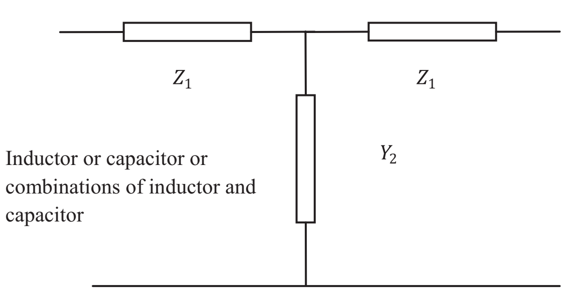

Figure 1 Transmission line model. (Unit Cell.)

spectral densities are confined within some reasonable frequency limits like as is defined in half power bandwidth for low pass or high pass filters.

For analyzing the non-linear circuit model, circuit theory has been used (KCL and KVL) [13] to come up with the relation between input and output quantities. At the input of the model, a Gaussian voltage pulse is applied and the output voltage is measured in time domain. Then these quantities are converted into frequency domain and along with the transmission line theory, the dispersion characteristics are obtained. Dispersion is the relation between phase constant and frequency. This can be calculated easily from the complex propagation constant as $\gamma = \sqrt{Z_1 Y_2} = \alpha + j\beta$, where α is attenuation constant and β is a phase constant which gives the dispersion relation.

2.1. Equations governing the system

For an inductor on iron core, the relationship between magnetic flux and current passing through the loop is related as

$$i(t) = \frac{N\phi(t)}{L} + A\phi(t)^3 \quad (1)$$

where N is the number of turns of the coil, A is the cross sectional area of the core and L is the self inductance. For a linear inductor the coefficient $A = 0$. Using the Faraday's law of induction along with Lenz's law, the induced voltage can be found out as

$$v(t) = -N\frac{d\phi(t)}{dt} \quad (2)$$

For a linear inductor the relation between induced voltage and current is related as

$$v(t) = L\frac{di}{dt} \quad (3)$$

For a non-linear device, it is little difficult to come up with a simple equation of $\phi(t)$ in terms of $i(t)$ not involving radical signs. If the values of A, N and L are known, then with the help of curve fitting it is easy to approximate the same function and $\phi(t)$ can be expressed as some function of $i(t)$. This is illustrated with the following numbers.

Fig. 2 represents this non-linear relation between flux and current with $N = 5$; $L = 1H$; $A = 0.1\ \mathrm{m}^2$ when flux is changing from 0 to 20 T. This non-linearity can be converted into an equation for $\phi(t)$ in terms of $i(t)$ with the help of curve fitting technique or interpolation. As though these numbers seem to be not practical at the present moment, there is no mistake in analyzing this model to study the behaviour of the circuits with non-linearities. From interpolation it is possible to get the following equation

$$\phi(t) = p_0 + p_1 i(t) + p_2 i(t)^2 + p_3 i(t)^3 + p_4 i(t)^4 \quad (4)$$

where $p_4 = -0.1776 \times 10^{-9}$, $p_3 = 0.3623 \times 10^{-6}$, $p_2 = -0.262 \times 10^{-3}$, $p_1 = 0.09147$ and $p_0 = 1.309$.

Using the above equation, it is easy to find the induced voltage using the equation $v(t) = -N\frac{d\phi(t)}{dt}$. This is considered in detail in next section to represent a close approximation to the practical application.

After observing that non-linearity, it is a reasonable choice to analyze the non-linear inductor with the following relationship between voltage and current as