Ain Shams Engineering Journal (2016) 7, 313–333

Ain Shams University

Ain Shams Engineering Journal

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MECHANICAL ENGINEERING

A homogenization procedure for geometrically non-linear free vibration analysis of functionally graded annular plates with porosities, resting on elastic foundations

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Received 3 December 2014; revised 9 September 2015; accepted 16 November 2015
Available online 14 January 2016

KEYWORDS

Non-linear vibration;
Annular plate;
Functionally Graded Material (FGM);
Porosities;
Homogenization procedure;
Elastic foundations

Abstract Some Functionally Graded Materials contain pores due to the result of processing; this influences their elastic and mechanical properties. Therefore, it may be very useful to examine the vibration behavior of thin Functionally Graded Annular Plates Clamped at both edges including porosities. In the present study, the rule of mixture is modified to take into account the effect of porosity and to approximate the material properties assumed to be graded in the thickness direction of the examined annular plate. A semi-analytical model based on Hamilton's principle and spectral analysis is adopted using a homogenization procedure to reduce the problem under consideration to that of an equivalent isotropic homogeneous annular plate. The problem is solved by a numerical iterative method. The effects of porosity, material property, and elastic foundations characteristics on the CCFGAP axisymmetric large deflection response are presented and discussed in detail.

1. Introduction

A lot of research and development have been carried out to investigate Functionally Graded Materials FGMs for various applications using gradients in physical, chemical, biochemical, and mechanical properties. The main characteristic that distinguishes FGMs from conventional composite materials is the possibility of tailoring the graded composition and microstructure in an intentional manner, destined to achieve the desired function. The design of FGM structures is based on an attempt to take the best benefit from the integration of the functions of

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Peer review under responsibility of Ain Shams University.

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http://dx.doi.org/10.1016/j.asej.2015.11.016
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Nomenclature

CCFGAP	Clamped–Clamped Functionally Graded Annular Plates
DQM, DSC	Differential Quadrature Method and Discrete Singular Convolution respectively
HDQ, FEM	Harmonic Differential Quadrature and Finite Element Method respectively
FSDT, FD	First order Shear Deformation Theory and Finite Differences respectively
b, a, h	inner radius, outer radius and thickness of the annular plate respectively
α	inner to outer radius ratio of the annular plate. $\alpha = b/a$
λ	thickness to outer radius ratio of the annular plate. $\lambda = h/a$
ϑ_c, ϑ_m	volume fraction of ceramic and metal respectively
k, ζ	gradient index and porosity volume fraction respectively
v	Poisson's ratio of the annular plate material
E_c, E_m	Young's modulus of ceramic and metal respectively
ρ_c, ρ_m	mass density of ceramic and metal respectively
(r, θ, z)	cylindrical co-ordinates
W, U	transverse and in-plane displacements of the middle plane point (r, θ, z) respectively
u_r, u_z	displacements along r and z directions respectively
$\varepsilon_r, \varepsilon_\theta$	radial and circumferential strains respectively
V_b, V_m, V	bending, membrane and total strain energy respectively
V_f, T	potential energy of the elastic foundation and Kinetic energy
K_L, K_{NL}, K_S	Winkler, non-linear and shear layer foundation parameters respectively
A_{11}, B_{11}, D_{11}	extensional stiffness, extensional–bending coupling stiffness and bending stiffness respectively
δ, β, γ	$\delta = B_{11}/A_{11}, \beta = \left(A_{11}h^2\right)/\overline{D_{11}}, \gamma = \left(A_{11}h^2\right)/4\overline{D_{11}}$
Mode (m, n)	m: nodal diameters. n: interior nodal circles
$k^w_{ij}, m^w_{ij}, b^w_{ijkl}$	general terms of the rigidity tensor, the mass tensor and the fourth order non-linearity tensor, respectively, associated with the transverse displacement
k^u_{ij}, m^u_{ij}	general terms of the rigidity tensor and the mass tensor, respectively, associated with the in-plane displacement
C^{uw}_{ijk}	general term of the third order non-linearity rigidity tensor representing the coupling between the in- and the transverse displacements
d^*_{ijk}	general term of the third order tensor allowing the calculation of the kth in-plane contribution coefficient
b^*_{ijkr}	general term of the fourth order non-linearity rigidity tensor taking into account the influence of the in-plane displacement
w^*_{max}	maximum non-dimensional vibration amplitude
$w(r)$	transverse shape function $W(r, t) = w(r)\cos(\omega t)$
$u(r)$	in-plane shape function $U(r, t) = u(r)cos^2(\omega t)$
a_i, b_i	contribution coefficient of the ith transverse and in-plane basic function respectively. $w(r) = a_i w_i(r), u(r) = b_i u_i(r)$
po, pi	number of transverse and in-plane basic functions respectively
$\{A\}$	column matrix of transverse contribution coefficients: $\{A\}^T = \left[a_1, a_2, \ldots, a_{po}\right]$
η_i	the ith transverse eigenvalue parameter for axisymmetric CCFGAP
ξ_i	the ith in-plane eigenvalue parameter for axisymmetric CCFGAP
$\left(\omega^*_{nl}/\omega^*_l\right)_i$	the ith non-dimensional frequency ratio
$\sigma_{br}, \sigma_{b\theta}$	radial and circumferential bending stresses
$\sigma_{mr}, \sigma_{m\theta}$,	radial and circumferential membrane stresses
σ_{tr}	radial total stress
*	star exponent indicates non-dimensional parameter

refractoriness, high wear resistance hardness, thermal shock resistance, and corrosion resistance of ceramics on the one hand as well as the high strength and toughness of metals on the other hand. This has led to a variety of structural applications [1]. FGMs that are made from a mixture of metal and ceramics are typically characterized by a smooth and continuous change of the mechanical, physical, and chemical properties from one side to the other [2–4]. These FGMs designs are reported to overcome weaknesses of laminated composite materials such as de-bonding under massive stress, local large plastic deformations or the buildup of residual stresses caused by the large difference in thermal expansion coefficients among components [5]. Generally speaking, the compositional gradient obtained in FGMs offers an excellent solution for such cases as a result of a continuous transition from one material property to the other avoiding abrupt mismatch [6], which keeps the thermo-mechanical stresses within acceptable limits and minimizes the residual thermal stresses [7]. FGMs have been widely used in aerospace, energy, electronics, and other industries. With the increasing attention of researchers, a wide variety of processes has been used in the preparation of FGMs such as powder metallurgy, vapor deposition, self-propagation, centrifugal casting, and magnetic separation [8–12]. All of these methods mentioned above have their own drawbacks such as high costs and complexity of the technique. The sintering process proves to be a flexible and suitable route for FGM manufacturing. However, during the process, microvoids or porosities can occur within the material, due to the large difference in the solidification temperatures between the material constituents [13]. The presence of small pores results in a sharp decrease in strength and elastic modulus. Hence, it is important to take into account the porosity effect when designing FGM structures subjected to dynamic loadings. To describe the interaction of a plate with its foundation, various basic models have