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Nonlinear throughflow effects on thermally modulated porous medium

Palle Kiran [a], B.S. Bhadauria [b,*]

[a] *Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow 226025, India*
[b] *Department of Mathematics, Faculty of Sciences, Banaras Hindu University, Varanasi 221005, India*

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Abstract Effect of vertical throughflow on Darcy convection has been investigated subject to time-periodic temperature modulation of the boundaries. The amplitudes of temperature modulation at the lower and upper surfaces are considered to be very small, and the disturbances are expanded in terms of power series of amplitude of convection. A weak nonlinear stability analysis has been performed for the stationary mode of convection, and heat transport in terms of the Nusselt number, which is governed by the non-autonomous Ginzburg–Landau equation, is calculated. The effect of vertical throughflow is found to be either to destabilize or stabilize the system for downward or upward throughflows in the case of impermeable boundary conditions. The effect of amplitude and frequency of modulation, Prandtl–Darcy number on heat transport has been analyzed and depicted graphically. Further, the study establishes that the heat transport can be controlled effectively by a mechanism that is external to the system.

1. Introduction

The buoyancy driven convection in fluid saturated porous media is of fundamental interest due to its practical applications such as geothermal energy utilization, enhanced recovery of petroleum reservoirs, insulation of reactor vessels, polymer engineering, ceramic processing and nuclear waste repositories, to mention a few. The enormous volume of work devoted to this field is well documented in the literature, Ingham and Pop [1], Nield and Bejan [2], Vafai [3]. Because of these applications, together with the fact that porous media occur in many natural situations, several studies have been undertaken to analyze the effects of different phenomena connected with such media. An excellent review of most of these studies has been reported in Nield and Bejan [4]. In the aforementioned applications, control of convective instability plays an important role. One of the effective mechanisms that control convective instability is that of maintaining a nonlinear temperature gradient. Recently, considering various convective flow models in porous medium [5–7], fluid layer [8–10] the phenomenon of heat or mass transfer investigated, where the concept of regulating either heat or mass

* Corresponding author. Tel.: +91 542 6702512.
E-mail addresses: kiran40p@gmail.com (P. Kiran), mathsbsb@yahoo.com (B.S. Bhadauria).
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transfer is missing. The temperature gradient can be achieved by time-dependent heating or cooling at the boundaries, the related problems have been investigated by Nield [11], Chhuon and Caltagirone [12], Rudraiah et al. [13], Rudraiah and Malashetty [14], Caltagirone [15], Bhatia and Bhadauria [16,17], Bhadauria [18–24], Bhadauria and Suthar [25], Bhadauria and Srivastava [26], Bhadauria et al. [27], Bhadauria and Kiran [28,29] and Kiran and Bhadauria [30].

However, several geophysical and technological applications involve non-isothermal flow of fluids through porous media, called throughflow (i.e., there is flow across the porous medium and the basic flows not quiescent). Such a flow alters the basic temperature profile from linear to nonlinear with layer height, which in turn affects the stability of the system significantly. The effect of throughflow on the onset of convection in a horizontal porous layer has been studied by Wooding [31], Sutton [32], Homsy and Sherwood [33], Jones and Persichetti [34]. Nield [35] and Shivakumara [36] showed that a small amount of throughflow can have a destabilizing effect, if the boundaries are of different types. Khalili and Shivakumara [37] have investigated the effect of throughflow and internal heat generation on the onset of convection in a porous medium. They have shown that throughflow destabilizes the system even if the boundaries are of the same type; a result which is not true in the absence of an internal heat source. The non-Darcian effects on convective instability in a porous medium with throughflow have been investigated in order to account for inertia and boundary effects by Shivakumara [38]. Shivakumara and Nanjundappa [39] investigated analytically, the effects of quadratic drag and vertical throughflow on double diffusive convection in a horizontal porous medium using the Forchheimer extended Darcy equation. It is found that, irrespective of the nature of boundaries, a small amount of throughflow in either of its direction destabilizes the system; a result which is in contrast to the single component system. Shivakumara and Sureshkumar [40] have studied convective instability in non-newtonian fluid saturated porous medium in the presence of vertical throughflow and found that throughflow has stabilizing or destabilizing effect depending on the boundaries and the directions of the flow. Brevdo [41] investigated three-dimensional absolute and convective instabilities at the onset of convection in a porous medium with inclined temperature gradient and vertical throughflow. Barletta et al. [42] analyzed the convective roll instabilities of vertical throughflow with viscous dissipation in a horizontal porous medium. The effects of hydrodynamic and thermal heterogeneity, horizontal throughflow on the onset of convection in a horizontal layer of a saturated porous have been investigated by Nield and Kuznetsov [43]. They found that the horizontal throughflow has no effect on the stability. When the permeability increases in the direction of the throughflow a small amount of throughflow may destabilize the transverse modes and so destabilize the layer as a whole. Reza and Gupta [44] investigated the effect of throughflow on the onset of convection in a horizontal layer of electrically conducting fluid, confined between two rigid permeable boundaries, and heated from below in the presence of a uniform vertical magnetic field. They found that magnetic field inhabits the onset of steady convection, and a positive throughflow is more stabilizing than negative throughflow. Patil and Rees [45], investigated the effects of local thermal nonequilibrium on the linear stability of the thermal boundary layer formed by a constant downward throughflow. They found that the basic temperature field is altered from the pure exponential form which arises when the phases are in LTE. They also found that, small values of either inter-phase heat transfer coefficient or the porosity-modified conductivity ratio cause the boundary layer to split into two distinct regions, an inner region, which arises because of the effect of the intrinsic suction velocity, and an outer region, which is due to the poor transfer of heat between the phases. Recently Nield and Kuznetsov [46], considering iso-flux and iso-temperature boundaries, investigated the effect on onset of convection in a layered porous medium with vertical throughflow and found that throughflow has a stabilizing effect whose magnitude may be increased or decreased by the heterogeneity.

From the above paragraph, it is observed that a huge amount of analysis on throughflow has been discussed on the onset of convection for various flow models. However, not much work has been done on throughflow considering nonlinear theory, which is essential to analyze the effect of heat transfer on the system. Further, to the best of authors' knowledge, not even a single study which considers linear/nonlinear thermal instability on throughflow under modulation is available in the literature. Therefore, in this paper, we intend to study, the effect of constant throughflow on Darcy convection, subjected to temperature modulation of the boundaries, by making a weak nonlinear stability analysis. The heat transport across the porous medium is quantified in terms of the Nusselt number, obtained by solving the non-autonomous Ginzburg–Landau equation.

2. Mathematical formulation

We consider an infinitely extended horizontal porous medium saturated by Newtonian fluid, confined between two free–free boundaries at $z = 0$ and $z = d$, and heated from below. The temperature of the boundaries varies periodically in a time-dependent manner. The temperature difference across the porous medium is kept at ΔT. We choose Cartesian frame of reference as, origin in the lower boundary and the z-axis in vertically upward direction. The schematic diagram is shown in Fig. 1, given below. It is assumed that the mechanical properties and thermal properties in x and y-directions are same. Further, Darcy law and the Oberbeck–Boussinesq approximation are considered. Under these assumptions, the equations which describe the system are given by the following:

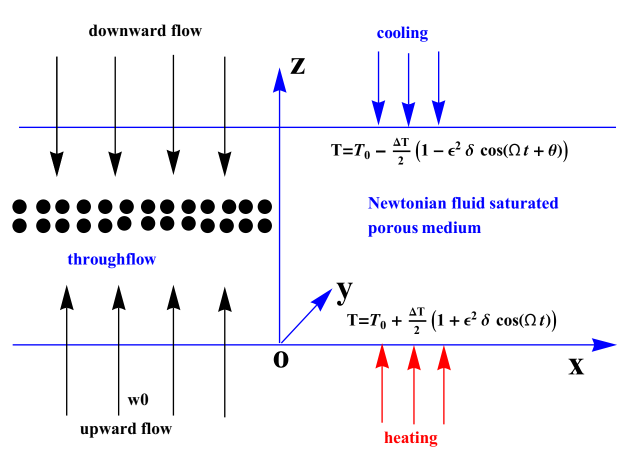

Figure 1 A sketch of the physical problem.