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Heat transfer analysis for squeezing flow of a Casson fluid between parallel plates

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Abstract Heat transfer analysis for the squeezing flow of a Casson fluid between parallel circular plates has been presented. Viable mathematical model has been constructed by using conservation laws coupled with suitable similarity transforms. This model ends up on a set of two highly nonlinear ordinary differential equations. Resulting equations have been solved by using a well-known analytical technique homotopy perturbation method (HPM). A numerical solution using forth order Runge–Kutta method has also been sought to support our analytical solution and the comparison shows an excellent agreement. Flow behavior under altering involved physical parameters is also discussed and explained in detail with graphical aid. For the presented problem, values of parameters are restricted. Analysis is carried out using the following ranges of parameters; squeeze number ($-4 \leqslant S \leqslant 4$), Casson fluid parameter ($0.1 \leqslant \beta \leqslant \infty$), Prandtl number ($0.1 \leqslant Pr \leqslant 0.7$), Eckert number ($0.1 \leqslant Ec \leqslant 0.7$) and $0.1 \leqslant \delta \leqslant 0.4$. Increase in velocity for squeeze number and Casson fluid parameter is observed. Temperature profile is found to be decreasing function of squeeze number and Casson fluid parameter and increasing function of Pr, Ec and δ.

1. Introduction

Heat transfer occurs in many physical situations. Moving bodies are heated not only due to some external source but their motion against other surfaces may also produce frictional heat which is of great interest. Sustainability of mechanical systems which consist of rapidly moving pistons or parts can be increased by proper understanding of heat transfer occurring in those systems. For proper working of these machines, lubricants are used to reduce the friction between parts. Rheological properties of these lubricants may vary under certain thermal conditions therefore for assembling highly efficient machines heat transfer analysis is very important.

Squeezing flow between orthogonally moving circular plates is involved in many practical situations. Its applications especially in polymer processing, modeling of synthetics transportation inside living bodies, hydro-mechanical machinery

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and compression/injection processes are of great importance. Several researchers have considered these flows and have contributed their effort for better understanding of such types of flows. The seminal contribution in this regard can be named to Stefan [1]. His pioneering effort has opened new doors for the researchers and lot of studies have been carried out following him [2–8]. Homotopy perturbation solution for two-dimensional MHD squeezing flow between parallel plates has been determined by Siddiqui et al. [9]. Domairry and Aziz [10] investigated the same problem for the flow between parallel disks. Recently, Mustafa et al. [11] examined heat and mass transfer for squeezing flow between parallel plates using homotopy analysis method (HAM).

In most of realistic models the fluids involved are not simple Newtonian. Complex rheological properties of non-Newtonian fluids cannot be captured by a single model. Different mathematical models have been used to study different types of non-Newtonian fluids. One of such models is known as Casson fluid model. Refs. [12,13] showed that it is most compatible formulation to simulate blood type fluid flow. It is clear from the literature survey that squeezing flow of Casson fluid between plates moving normal to their own surface is yet to be inspected.

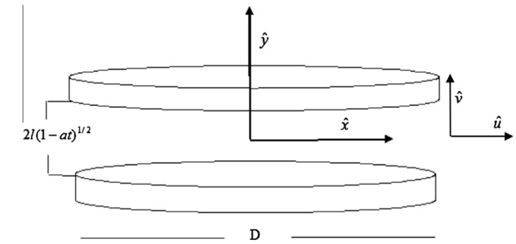
Due to inherent nonlinearity of the equations governing fluid flow most of the problems lack exact solutions. Even where exact solutions are available immense simplification assumptions have been imposed. Those overly imposed suppositions may not be used for more realistic flows. However to deal with this hurdle many analytical approximation techniques have been developed which are commonly used nowadays.

Solution to aforementioned highly nonlinear ordinary differential equation still lacks exactness due to its abstract nature. However, different attempts have been made to approximate its solution in an acceptable and accurate way. Nowadays, several approximation techniques have been developed to fulfill this purpose [14–31]. From them, those are used most often which are easy to apply and require less computational work yet provide reliable results.

Here we present heat transfer analysis for the squeezing flow of a Casson fluid between parallel plates. Mathematical form of the problem is extracted by using conservation laws along with similarity transformations. Resulting highly nonlinear ordinary differential equation is then solved by using homotopy perturbation method. This method has successfully been used by numerous researchers in different scientific problems [32–41]. One can also see from our work that the obtained analytical solution shows excellent compatibility with numerical solution obtained by Runge–Kutta order 4 [RK-4] method.

2. Governing equations

Consider an incompressible flow of a Casson fluid between two parallel plates distance $z = \pm l(1-\alpha t)^{1/2} = \pm h(t)$ apart, where l is the initial position (at time $t = 0$). Further, $\alpha > 0$ corresponds to squeezing motion of both plates until they touch each other at $t = 1/\alpha$, for $\alpha < 0$ the plates leave each other and dilate (Scheme 1). Also, viscous dissipation effects are retained to study the generation of heat due to friction caused by shear in the flow. Rheological equation of Casson fluid is defined as under [42–46]



Scheme 1 Schematic diagram of the flow problem.

$$\tau_{ij} = \left[\mu_B + \left(\frac{p_y}{2\pi}\right)\right] 2e_{ij}, \tag{1}$$

μ_B is dynamic viscosity of the non-Newtonian fluid, p_y is yield stress of fluid and π is the product of component of deformation rate with itself, *i.e.* $\pi = e_{ij}e_{ij}$, where e_{ij} is the (i, j)th component of the deformation rate.

The equations governing the flow are:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0, \tag{2}$$

$$\begin{aligned} \frac{\partial \hat{u}}{\partial t} + \hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} &= -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{x}} + \upsilon\left(1+\frac{1}{\beta}\right)\left(2\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{y}\partial \hat{x}}\right), \\ \frac{\partial \hat{v}}{\partial t} + \hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} &= -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{y}} + \upsilon\left(1+\frac{1}{\beta}\right)\left(2\frac{\partial^2 \hat{v}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{y}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{y}\partial \hat{x}}\right), \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{\partial T}{\partial t} + \hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} &= \frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial \hat{x}^2} + \frac{\partial^2 T}{\partial \hat{y}^2}\right) \\ &+ \frac{v}{C_p}\left(1+\frac{1}{\beta}\right)\left(2\left(\frac{\partial^2 \hat{u}}{\partial \hat{x}^2}\right)^2 + \left(\frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{x}^2}\right)^2 + 2\left(\frac{\partial^2 \hat{v}}{\partial \hat{y}^2}\right)^2\right). \end{aligned} \tag{4}$$

In the above equations $\hat{u}$ and $\hat{v}$ are the velocity components in $\hat{x}$ and $\hat{y}$-directions respectively, $\hat{p}$ is the pressure, T is the temperature, v is the kinematic viscosity of the fluid and $\beta = \mu_{\mathbf{B}}\sqrt{2\pi_c}/p_y$ is the Casson fluid parameter. Also, ρ is density of the fluid, C_p is the specific heat and k is thermal conductivity of the fluid. Viscosity of the fluid is taken as constant and it does not depend on temperature.

Boundary conditions for the flow problem are

$$\begin{aligned} &\hat{u} = 0, \quad \hat{v} = v_w = \tfrac{dh}{dt}, \quad T = T_H, \quad at\ \hat{y} = h(t), \\ &\tfrac{\partial \hat{u}}{\partial \hat{y}} = 0, \quad \tfrac{\partial T}{\partial \hat{y}} = 0, \qquad \hat{v} = 0, \quad at\ \hat{y} = 0. \end{aligned} \tag{5}$$

We can simplify this system of equations by eliminating pressure terms from Eqs. (2) and (3) and using Eq. (1). After cross differentiation and introducing vorticity ω, we get

$$\frac{\partial \omega}{\partial t} + \hat{u}\frac{\partial \omega}{\partial \hat{x}} + \hat{v}\frac{\partial \omega}{\partial \hat{y}} = \upsilon\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 \omega}{\partial \hat{x}^2} + \frac{\partial^2 \omega}{\partial \hat{y}^2}\right), \tag{6}$$

where

$$\omega = \left(\frac{\partial \hat{v}}{\partial \hat{x}} - \frac{\partial \hat{u}}{\partial \hat{y}}\right). \tag{7}$$

Using transform introduced by [11,14] for a two-dimensional flow

$$\hat{u} = \frac{\alpha \hat{x}}{[2(1-\alpha t)]} F'(\eta), \tag{8}$$