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New and improved hydraulic radius for channels of the second kind

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Abstract The Manning's equation is commonly used to calculate discharge and mean velocity of the uniform flows. According to experimental data, Manning's equation with constant Manning's coefficient overestimates the discharge of second-kind channels (channels with a closing top-width), under the partially full flow. This problem can be solved by altering the Manning's coefficient depending on the relative flow depth or changing the definition of the conventional hydraulic radius, that is, flow area divided by the wetted perimeter. Since, Manning's coefficient theoretically depends only on the materials of the wall, so it seems that the second method is preferable.

In current research a new and improved definition of hydraulic radius for closed conduits flowing partially full, is presented. This definition is efficient enough and provides powerful tool to determine the channel discharge and friction slope of uniform flow via Manning's equation.

1. Introduction

In sewer and tunnel design, it is necessary to be able to predict the friction slope and velocity or discharge when channel cross-section is partly full. Manning's equation has been the most commonly used formula in sewer and tunnel design because of its simplicity and the generally satisfactory results [1]. This equation is still used as a standard equation for calculation of mean velocity in uniform flow and hydraulically rough channels with various cross-sectional shapes. Manning's equation uses roughness coefficient, n, to represent resistance effects. It is often found in the form

$$V = \frac{K_n}{n} R_h^{2/3} S^{1/2} \tag{1}$$

where V is velocity, n is Manning's resistance coefficient, R_h is conventional hydraulic radius (flow area divided by the wetted perimeter), S is friction slope and K_n is a constant equal to 1 for metric units and 1.486 for English units.

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Notation

A	area of flow
A_E	effective flow area
D	diameter/height of the channel
e	height of the bottom arc of horseshoe cross-section
f_s	free-surface weight factor
h	flow depth in the horseshoe channel
K_n	conversion coefficient
$l(x, y, \theta)$	distance between point (x, y) within the conduit and point on wall designated by angle θ
$l(x, y, i)$	distance between point (x, y) within conduit and a point on wall designated by index i
$l(x, y, j)$	distance between point (x, y) within conduit and point on free surface designated by index j
$L_H(x, y)$	harmonic mean of distances at point (x, y)
m	a dummy dimensionless parameter
N	division number
n	Manning's resistance coefficient
N_1	division number selected to wall region
N_2	division number selected to free-surface region
P_T	total flow perimeter including top water surface
Q	flow rate
R	radius of horseshoe cross-section
R_E	effective radius
R_h	conventional hydraulic radius
R_h	harmonic radius
R_H	hydraulic radius
R_{HE}	effective hydraulic radius
S	friction slope
t	a characteristic parameter $(= R/r)$
V	velocity
x_*	dimensionless variable $(= x/D)$
y_*	dimensionless variable $(= y/D)$
θ	angle designating direction of $l(x, y, \theta)$
ε	dimensionless variable $(= e/D)$
η	dimensionless variable $(= h/D)$
λ	dimensionless variable $(= x_t/D)$
ΔP	difference of the total flow perimeter$(= P_T/N)$

It is worth noting that the channel surface roughness, ε, for establishing the hydraulically rough flow should be greater than $30v\,[Q(gS)^2]^{-0.2}$, where v, g and Q are the kinematic viscosity, gravitational acceleration and flow discharge, respectively [2]. Moreover, Manning's equation is valid for $0.004 \leqslant \varepsilon/R \leqslant 0.04$, where R is the hydraulic radius defined as the ratio of the flow area to the flow perimeter [3]. The Manning's equation is usually used by assuming that the roughness coefficient is constant [4,5]. However, according to Camp's experimental data, Manning's equation with constant Manning's coefficient overestimates the discharge of circular channels under the partially full flow. He attributed this issue to assuming constant Manning's coefficient for all flow depths which is not a realistic assumption. He presented experimental data in graphical form showing the variation of relative velocity and relative discharge as a function of the relative flow depth. These charts have practical usage and have been reproduced in many widely used texts (e.g., [4–6]). These curves are also accepted by the profession and are reproduced in manuals by the American Society of Civil Engineers [7].

To generalize the Manning's equation, Fukuchi (2006) [8] using a constant manning's coefficient, proposed the concept of harmonic mean distances from a wall for a new definition of hydraulic radius. The results of Fukuchi's model showed good agreement with Camp's chart. He further argued that his new definition of hydraulic radius can also be applied to flow in simple cross-sections and may be developed further for use with compound channel flows. It seems his model has a potential to be modified and used to horseshoe cross-sections which are very similar to circular cross-sections.

It is important to note that a channel of the second kind such as circular cross-section channel is a special case of horseshoe cross-section channels. Because of superior structural and hydraulic characteristics of horseshoe cross-sections, these shapes of the channel are frequently used for free-surface water conveyance tunnels [9]. Standard horseshoe cross-sections have been designed and built for many hydraulic projects in different countries [10–12].

As described in next sections, Fukuchi's model which is presented in *polar coordinates* is difficult for integration procedure. His definition for hydraulic radius also yields an unnecessary constant for the model, which made it hard to use for different cross sections.

In current research motivated by Fukuchi's idea, a new hydraulic radius is defined in *Cartesian coordinates*. In this way the unnecessary constant will be eliminated and integration procedure needed in the model will be simplified significantly. This new model is calibrated using Camp's experimental data for circular cross section and then applied to all types of the standard horseshoe cross-sections which need to be verified using the experimental data.

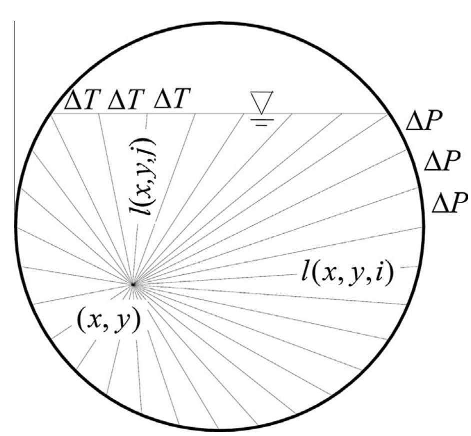

Figure 1 Schematic sketch for calculation of effective hydraulic radius, R_{HE}, in terms of $l(x, y, i)$ and $l(x, y, j)$.