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Probing quantum spin glass like system with a double quantum dot

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ABSTRACT

We study the ground state properties of a 4-qubit spin glass like (SGL) chain with probes at the end of the chain and compare our results with the non-spin glass like (NSGL) case. The SGL is modeled as a spin chain with nearest-neighbor couplings, taking on normal variates with mean J and variance Δ^2. The entanglement between the probes is used to detect any discontinuity in the ground state energy spectrum. For the NSGL case, it was found that the concurrence of the probes exhibits sharp transitions whenever there are abrupt changes in the energy spectrum. In particular, for the 4-qubit case, there is a sudden change in the ground state energy at an external magnetic field B of around 0.66 (resulting in a drop in concurrence of the probes) and 1.7 (manifest as a spike). The latter spike persists for finite temperature case. For the SGL sample with sufficiently large Δ, however, the spike is absent. Thus, an absence in the spike could act as a possible signature of the presence of SGL effects. Moreover, the sudden drop in concurrence at $B \approx 0.66$ does not disappear but gets smeared with increasing Δ. However, this drop can be accentuated with a smaller probe coupling. The finite temperature case is also briefly discussed.

1. Introduction

Phase transitions in crystalline or ordered solids have been studied extensively and it is now quite well understood. However, this is not the case for disordered solids. One such disordered solid example is the spin glass, which remains as one of the most challenging and interesting problems in condensed matter physics. A spin glass consists of spins or magnetic moments arranged in randomness either through their positions or the different in signs between the neighboring couplings [1–7]. The neighboring couplings refer to ferromagnetic and antiferromagnetic. The disorder in site or coupling causes frustration in a spin glass. At certain critical temperature or sometimes called freezing temperature, this random collection of spins will be frozen in place. It is because of this frozen state transition that the frustration arises. Due to this frustration among the neighboring spins, many possible ground state configurations resulted. This produced a complex and rugged free energy landscape.

Spin glasses are magnetic alloys comprising x concentration of magnetic impurities occupying random sites in a non-magnetic host metal [6]. By controlling the concentration and distribution of such impurities randomly on the host, it is possible to observe the spin glass transition at low temperature. Examples of such impurities include Manganese (Mn), Iron (Fe) and Europium (Eu). Magnetic alloys which are originally studied are $Cu_{1-x}Mn_x$ [8] and $Au_{1-x}Fe_x$ [9]. These magnetic alloys are considered as canonical spin glasses. Other alloys which are studied as a spin glass include $Eu_xSr_{1-x}S$ [10] and $La_{1-x}Gd_xAl_2$ [11]. Theories like the Edwards-Anderson (EA) model [12] which only allows the spins to interact via nearest-neighbor couplings with no long range order and Sherrington–Kirkpatrick (SK) model [13] for which every spin couples equally with every other spin are used in an attempt to explain mainly the cusp in the magnetic susceptibility. Essentially, the EA model replaces the site disorder and Ruderman–Kittel–Kasuya–Yosida (RKKY) distribution [14–16] with a random set of bonds, for instance, based on a gaussian distribution. An order parameter q is used to characterize the spin glass phase. The original EA equations are not simple to solve and are only soluble in the limits $T \to 0$ and $T \to T_f$. At these limits, the EA equations showed an asymmetric cusp in the magnetic susceptibility and specific heat. In contrast, the results by Fischer [17] showed that the theoretical specific heat is different from the experimental result except for the low temperature linear dependence when using spin $S = \frac{1}{2}$. Although the SK model did exhibit a cusp in the magnetic susceptibility and specific heat, the entropy S goes to negative limit and the free energy is maximum with respect to the order parameter q. In addition, when $q = 0$, the spin glass state has

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lower free energy than it is when $q \neq 0$. All these are in contrast with what a second order transition should exhibit. Due to this instability in SK solution, Almeida and Thouless (AT) [18] did a detailed analysis on it and showed that the solution is unstable at low temperature. They showed the stability limits of the SK solution with the AT line dividing the unstable and stable areas in the spin glass phase diagram. Primarily, the instability is due to the fact that SK model treats all the replicas as indistinguishable – a replica symmetric solution. In order to solve this problem, Parisi [19–23] came out with a replica symmetry breaking (RSB) scheme which removes the unphysical negative entropy. It was found to be at least marginally stable. In spite of some success in using these models to describe some behaviors of the spin glass, they are not able to fully account for all the experimental results. This may due to the fact that these theories are classical in nature and did not consider the quantization of the spins of the impurities [24]. Experimentally, the measurement of the alternating-current (a.c.) susceptibility on a spin glass exhibits a cusp at certain freezing temperature T_f at low applied magnetic field. The sharp cusp in the magnetic susceptibility suggests that the alloy undergoes a phase transition [25]. Indeed, for alloys like gold–iron [9] and copper–manganese [8], these spin glasses show a cusp at certain critical temperature. It has been shown that the produced cusp is sensitive to the applied magnetic field and with just 100 G of magnetic field, a broader maxima is produced [9,26,27]. Besides been field dependent, certain spin glasses are also found to be frequency dependent [8,24]. Other measurement like finding the specific heat of $Au_{0.92}$ $Fe_{0.08}$ [28] and CuMn [29] were studied. In recent years, $LiHo_xY_{1-x}$ F_4 which can be described with a quantum Ising spin glass model has been studied experimentally and numerically [30–35]. For an x concentration of ≤ 0.25, it is believed that there is a spin glass phase. As the concentration is diluted, it is still an open question of whether a spin glass or an antiglass spin phase exists.

In quantum information theory, the study of entanglement in spin chain is believed to be important [36–43] as it is regarded as a key resource in applications like quantum key distribution, quantum teleportation, quantum dense coding, entanglement swapping and others [41]. Quantum correlation have been used in many-body systems to explore the phase transition at zero and finite temperature [43]. In the past, different order parameters and excitation spectrum are been used to characterize the phase transition of a many-body system. The synergy between quantum information and condensed matter physics has provided many new insights and directions. Specifically, a thorough analysis of the entanglement in the quantum critical models have been done [44–46]. With the use of entanglement from quantum information theory, the study of the ground state for many-body systems reveals new and interesting properties. The appearance of abrupt changes in the entanglement in the quantum phase transition is seen as a sensitive probe in determining the phase transition in a spin chain when one external qubit is coupled to both sides of the chain. This approach has been applied to the *XY* Heisenberg model with the use of nonlocal probe qubits to detect the quantum phase transition [47–49]. The entire spin chain, including the nonlocal probe qubits can be engineered experimentally using quantum dots with one or more electrons [50–54]. Recently, Shim et al. [55] have studied the quantum phase transition by using a double quantum dot coupled locally to a *XXX* Heisenberg model spin chain as an alternative and efficient probe for detection of a quantum phase transition. Although much studies have been done on understanding quantum dots and using them as a probe to detect phase transition through computation of the entanglement between probes, very little work is found on understanding how the entanglement of the SGL chain behaves at low temperature when it is coupled to two external probes. Hence, the focus of this paper is to investigate the entanglement of a SGL *XXX* Heisenberg model by using nonlocal probe coupled to both ends of the SGL chain.

With this motivation, we numerically investigate a quantum SGL chain consisting of 4 qubits described by the *XXX* Heisenberg model. Each end of the SGL chain is then coupled to a qubit or quantum dot which serves as a probe. Using this model, we examine how the entanglement of the probes that are attached to the SGL chain changes by varying the external magnetic field and the standard deviation of the random coupling (between the SGL sites).

The paper is organized as follows. We begin in Section 2 by defining the Hamiltonian of a 4-qubit system coupled to a probe qubit on each end of the spin chain. The *XXX* Heisenberg model spin chain is essentially modeled as a SGL exhibiting the usual characteristics of disorder and frustration. We then discuss the entanglement (concurrence) between the 2 probes which is considered as nonlocal, after tracing out the SGL chain as a function of the external applied magnetic field and the standard deviation in the coupling numerically. These results are presented and discussed in Section 3. In Section 4, we summarize our results.

2. Theoretical formulation

In this section, we present the Hamiltonian model used in our study. After formulating the Hamiltonian for the SGL chain, we coupled both ends of the SGL chain with a probe (qubit). In order to look at the quantum correlation between the two probes, we trace out the SGL chain which can be in general make up of n sites and compute with suitable entanglement measure to observe the ground state properties of the SGL chain. The *XXX* Heisenberg SGL chain is described by

$$H_n = \sum_{i=1}^{n} J_i (S_i^\alpha S_{i+1}^\alpha) \tag{1}$$

where J_i are the random variables and $S_i^\alpha = \frac{\hbar}{2}\sigma_i^\alpha$ denotes the Pauli matrices $(\alpha = x, y, z)$ of the ith spin [43] which is subject to the open boundary condition $S_{i+1}^\alpha \neq S_1^\alpha$. In this case, the ith spin represents the individual site number from the SGL chain. The exchange energies J_i are quenched random variables with a probability distribution $P(J_i) = \frac{1}{\sqrt{2\pi\Delta}} e^{-J^2/2\Delta^2}$ where Δ is the standard deviation for the distribution. By applying a uniform external magnetic field on each qubit in the SGL chain, the Hamiltonian for the SGL chain (H_{sgl}) model is given by

$$H_{sgl} = \sum_{i=1}^{n} J_i (S_i^\alpha S_{i+1}^\alpha) + B \sum_{i=1}^{n} S_i^z \tag{2}$$

where B is the external magnetic field applied transversely across the individual spin site S_i^z. By coupling a single probe (qubit) at each end of the SGL chain,

$$H_{sglp} = \sum_{i=1}^{n} J_i (S_i^\alpha S_{i+1}^\alpha) + B \sum_{i=1}^{n} S_i^z + J_p (S_1^\alpha S_{p1}^\alpha + S_N^\alpha S_{p2}^\alpha) \tag{3}$$

where $S_{p_1}^\alpha$ and $S_{p_2}^\alpha$ are the spin matrices $(\alpha = x, y, z)$ for the probes at the start and end of the chain respectively. Experimentally, the probe could be implemented with quantum dots. The probe coupling J_p describes the bond between the probes (q_{p1} and q_{p2}) and end sites of the SGL chain (q_1 and q_n) respectively. The SGL chain is described with the qubit sites as $q_1, q_2, \ldots q_{n-1}, q_n$. The model is illustrated in Fig. 1.

In order to measure the quantum entanglement between the two nonlocal probes, we adopt a bipartite measure for a two-level system called concurrence [56,57]. It is a measure of the non-separability of two-qubit density matrix with a value of zero for a