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Influences of Hall current and chemical reaction in mixed convective peristaltic flow of Prandtl fluid

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The objective of present analysis is to address the mixed convective peristaltic flow of Prandtl fluid in a planar channel with compliant walls. Effects of applied magnetic field and Hall current are retained. Heat transfer in fluid flow is characterized through convective boundary conditions. Impact of first order chemical reaction together with Soret effect is examined. Problems formulation in view of long wavelength and low Reynolds number consideration is developed. The graphs are obtained numerically for the velocity, temperature, concentration and heat transfer coefficient. Results for Hall parameter and Hartman number on velocity have opposite characteristics.

1. Introduction

Peristalsis is renowned for the mixing as well as transporting of physiological fluids through contraction or expansion of an extensible tube containing liquid. It is a key mechanism for urine transport in ureter, carry of bile in bile ducts, chyme movement in intestinal tract, transfer of spermatozoa in cervical canal and so on. The activity is also responsible for technical roller and finger pumps operations and biomedical engineering processes. In the literature several investigations on the peristaltic transport of non-Newtonian fluids under the assumptions of large wavelength and low Reynolds number are presented. Some investigations on the topic can be seen reported in the studies [1–9].

It is known that mixed convection occurs when both free and forced convection takes part in heat transfer process. This effect is utilized in several scientific and technological fields such as geology, biology, astrophysics and chemical processes. In translocation of water in tall trees, dynamic of lakes, solar ponds, lubrication and drying technologies, diffusion of nutrients out of blood, oxygenation, hemodialysis and nuclear reactors the importance of convective heat exchange with peristalsis cannot be under estimated. Thus consequences of mixed convective peristaltic transport of nanofluid in presence of Soret and Dufour effects is studied by Hayat et al. [10]. Mustafa et al. [11] investigated the numerical solution for mixed convective peristaltic flow of fourth grade fluid. Influence of heat and mass transfer in terms of mixed convection, initial stress and radially varying magnetic field on the peristaltic flow in an annulus has been discussed by Abd-Alla et al. [12]. Effects of mixed convection flow of nanoparticles on the peristaltic motion of tangent hyperbolic fluid model in an annulus has been studied by Nadeem et al. [13]. Tripathi and Beg [14] studied peristaltic flow of nanofluids in drug delivery systems. Peristaltic transport of magneto-nanoparticles submerged in water for drug delivery system is discussed by Abbasi et al. [15]. Hayat et al. [16] discussed an inclined magnetic field on peristaltic flow of Williamson fluid in an inclined channel with convective conditions and mixed convection. Later in another work [17] they investigated the simultaneous effects of convective conditions and nanoparticles on peristaltic motion.

The fluid flows in the presence of magnetic field has promising applications in engineering, chemistry, physics, the polymer industry and metallurgy. Some examples include controlling the rate of cooling, blood plasma, drying, evaporation at the surface of a water body, geothermal reservoirs, thermal insulation, enhanced oil recovery, cooling of nuclear reactors, bleeding reduction during surgeries, hyperthermia etc. Also under the influence of powerful applied magnetic fields the Hall effects in peristalsis cannot be ignored. In such case the applied magnetic field is powerful or collision frequency is small. In view of this Hayat et al. [18] discussed MHD peristaltic flow of a Maxwell fluid in a porous medium numerically. They used modified Darcy's law in presence of Hall effect

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which approaches to good agreement. Further they investigated the numerical analysis for MHD peristaltic transport of Carreau–Yasuda fluid in a curved channel with Hall effects [19]. Their results show balance of the magnetic influence by Hall parameter to some extent. Abbasi et al. [20] examined peristalsis of silver-water nanofluid in the presence of Hall and Ohmic heating effects. Results indicate that presence of these effects lessens the changes brought by an applied magnetic field in the state of nanofluids. Simultaneous effects of Hall current and homogeneous/heterogeneous reaction on peristalsis is studied by Hayat et al. [21]. Effect of Hall currents on interaction of pulsatile and peristaltically induced flows of a particle–fluid suspension is investigated by Gad [22]. Further he made extension to his work with the consideration of Hall effect and compliant wall properties to obtain qualitatively better results than the previous approach [23].

Motivated by the above mentioned applications the aim here is to study the effects of Hall current in mixed convection flow of Prandtl fluid in a channel with convective boundary conditions and compliant wall properties. In addition the Soret and chemical reaction effects are considered. To our knowledge, no study has been undertaken yet to investigate such salient features in peristalsis. No doubt the chemical reaction is significant in food processing, chemical processing, crops damage via freezing temperature distribution and groves of fruit trees. Thus relevant equations are modeled and then reduced to be simplified by lubrication approach. Results for stream function, temperature, concentration and heat transfer coefficient are obtained graphically and argued physically.

2. Problems development

We consider the mixed convective peristaltic flow of an incompressible Prandtl fluid in a symmetric channel of thickness $2d$. The wave is propagating along the compliant walls of channel with wave speed c. Let $u(x, y, t)$ and $v(x, y, t)$ represent the components of velocity in the axial x and transverse y directions respectively. The peristaltic wave shape is represented by

$$y = \pm\eta(x, t) = \pm\left[d + a\sin\frac{2\pi}{\lambda}(x - ct)\right], \tag{1}$$

where a is the amplitude of wave, λ the wavelength and t the time. The displacements of the left and right walls are represented by $\pm\eta$ respectively. Further a uniform magnetic field $\mathbf{B} = (0, 0, B_0)$ with strength B_0 is applied. Electric field is absent. Hall effect is taken into account. Through the generalized Ohm's law in this case one can write

$$\mathbf{J} = \sigma[\mathbf{V}\times\mathbf{B} - m(\mathbf{J}\times\mathbf{B})], \tag{2}$$

in which $m = 1/n_e e$ is the Hall factor, n_e the mass of electron, e the charge of electron. Above equation can be solved in $\mathbf{J}$ to yield the Lorentz force in the form:

$$\mathbf{J}\times\mathbf{B} = -\frac{\sigma B_0^2}{1+m^2}[(u - mv)i + (mu + v)j], \tag{3}$$

where $m = \sigma B_0 m^*$ is Hall parameter, $\mathbf{J}$ represents the current density and σ the electrical conductivity.

The governing equations in presence of body forces are given by

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{4}$$

x-component of momentum equation comprising mixed convection effects:

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \frac{\sigma B_0^2}{1+m^2}u + \rho\beta_T g(T - T_0) + \rho\beta_C g(C - C_0). \tag{5}$$

y-component of momentum equation:

$$\rho\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \frac{\partial S_{yx}}{\partial x} + \frac{\partial S_{yy}}{\partial y}. \tag{6}$$

Energy equation comprising viscous dissipation effects:

$$\rho c_p\frac{\partial T}{\partial t} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \mathbf{L}.\,\mathbf{S}, \tag{7}$$

Concentration equation with Soret effect and chemical reaction:

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{Dk_T}{T_m}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - k_1(C - C_0). \tag{8}$$

Here for Prandtl fluid

$$S_{xy} = \frac{A\sin^{-1}\left[\frac{1}{C}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right]^{1/2}\right]}{\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right]^{1/2}}\frac{\partial u}{\partial y}. \tag{9}$$

In the above $\mathbf{L} = \text{grad}\,\mathbf{V}$, ρ the density of fluid, μ the fluid dynamic viscosity, ν the kinematic viscosity, T the fluid temperature, C the concentration of fluid, T_0 and C_0 the temperature and concentration at the lower and upper walls respectively, c_p the specific heat at constant pressure, g the gravitational acceleration, β_T and β_C the coefficients of thermal and concentration expansions respectively, D the mass diffusivity coefficient, k_T the thermal diffusion ratio, k the thermal conductivity, k_1 the chemical reaction rate constant, T_m the mean fluid temperature, $\mathbf{S}$ extra stress tensor for Prandtl fluid and A, C are the material constants of Prandtl fluid model.

The convective boundary conditions for the exchange of heat and mass, no slip condition and compliant nature of the walls are described through the expressions

$$k\frac{\partial T}{\partial y} = -h_1(T - T_0), \quad \text{at } y = \eta, \tag{10}$$

$$k\frac{\partial T}{\partial y} = -h_1(T_0 - T), \quad \text{at } y = -\eta, \tag{11}$$

$$u = 0, \quad v = \pm\eta_t \quad \text{at } y = \pm\eta, \tag{12}$$

$$k\frac{\partial C}{\partial y} = -h_2(C - C_0), \quad \text{at } y = \eta, \tag{13}$$

$$k\frac{\partial C}{\partial y} = -h_2(C_0 - C), \quad \text{at } y = -\eta, \tag{14}$$

$$\left[-\tau\frac{\partial^3}{\partial x^3} + m_1^*\frac{\partial^3}{\partial x\partial t^2} + d'\frac{\partial^2}{\partial t\partial x}\right]\eta = -\rho\frac{\partial u}{\partial t} + \frac{\partial S_{xy}}{\partial y} + \frac{\partial S_{xx}}{\partial x} + \rho\beta_T g(T - T_0) + \rho\beta_C g(C - C_0) - \frac{\sigma B_0^2}{1+m^2}u, \quad \text{at } y = \pm\eta, \tag{15}$$

with $h_1$and h_2 as the heat and mass transfer coefficients at the upper and lower walls of the channel respectively, τ the elastic tension in the membrane, m_1^* the mass per unit area and d' the

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