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Heat transfer analysis due to an unsteady stretching/shrinking cylinder with partial slip condition and suction

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Abstract This article deals with the laminar flow of a viscous fluid and heat transfer analysis due to a porous stretching/shrinking cylinder with partial slip condition. The flow equations corresponding to momentum and energy equations are transformed into a set of highly nonlinear ordinary differential equations by means of similarity transformations, which are then, solved numerically using Runge–Kutta–Fehlberg method. The physical significance of the various involved parameters on the flow velocity and temperature distribution is discussed through graphs and tables in detail. It is found that the dual solutions exist for the shrinking cylinder, while a unique solution exists for stretching cylinder. Comparison of the present results with the existing previous results is given and found to be in good agreement.

1. Introduction

The flow caused by stretching boundary arises frequently in materials manufactured by extrusion, glass-fiber and paper production. Following the pioneering work of Crane [1], recently the flow and heat transfer characteristics past a stretching/shrinking sheet with and without slip condition at the surface were studied by many authors [2–6]. On the other hand, Wang [7] was the first who discussed the steady flow of a viscous fluid outside of a stretching hollow cylinder by considering the ambient fluid at rest. Ishak et al. [8] studied the MHD flow and heat transfer analysis over a stretching cylinder. Again Ishak et al. [9] discussed the effects of uniform suction/blowing on the flow and heat transfer due to a stretching cylinder numerically using the Keller-box method. Ishak and Nazar [10] presented the laminar boundary layer flow of a viscous fluid along a stretching cylinder and obtained the similarity solution. Abbas et al. [11] investigated the laminar

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MHD flow and heat transfer of an electrically conducting viscous fluid outside a stretching cylinder in the presence of thermal radiation in a porous medium. Munawar et al. [12–14] reported the analytical and numerical results to discuss the unsteady boundary layer flow of a viscous fluid with/without heat transfer analysis over an oscillatory stretching cylinder. Zaimi et al. [15] studied the unsteady viscous flow over a shrinking cylinder with mass transfer. In another study, Zaimi et al. [16] demonstrated the unsteady flow and heat transfer of a nano-fluid over a contracting cylinder.

In most of the investigations, no-slip boundary condition (the assumption that a liquid adheres to a solid boundary) is established and $Kn = 0$, but in some situations such as emulsions, suspensions, foams and polymer solution [17], the no-slip condition is not adequate. For the range of $0.01 < Kn < 0.1$ (slip flow) the standard Navier–Stokes and energy equations can still be used by taking into account velocity slip. In recent years, the slip-flow regime has been widely studied and researchers have been concentrating on the analysis of micro-scale in the micro-electro-mechanical systems (MEMS) associated with the embodiment of velocity slip. Because of the micro-scale dimensions, the slip flow greatly differs from the traditional no-slip flow [18–20]. Sparrowet al. [21] considered the fluid flow due to the rotation of a porous surfaced disk and also employed a set of linear slip flow conditions. A substantial reduction in torque occurred as a result of surface slip. Wang and Ng [22] investigated the slip flow due to a stretching cylinder. They found that the slip greatly reduces the magnitudes of the velocities and the shear stress. Mukhopadhyay [23] investigated the chemically reactive solute transfer in a boundary layer flow along a stretching cylinder with partial slip condition. Recently, Mukhopadhyay [24] presented the analysis for the axisymmetric laminar boundary layer flow of a viscous fluid and heat transfer over a stretching cylinder under the influence of a uniform magnetic field and partial slip condition.

In the present article, we derive numerical solutions for the laminar flow of a viscous fluid and heat transfer analysis due to a porous stretching/shrinking cylinder in the presence of boundary slip condition. Similarity transformations are employed to render the nonlinear dimensional partial differential boundary layer equations into set of ordinary differential equations, which are solved using Runge–Kutta–Fehlberg method. The graphs are plotted and discussed for the variations of different involved parameters in detail.

2. Flow equations

Consider an unsteady, two-dimensional and incompressible flow of a viscous fluid over a permeable stretching/shrinking cylinder, where the z-axis is taken along the axis of cylinder and r-axis in radial direction as shown in Fig. 1. It is also assumed that the diameter of a cylinder is taken as a function of time with unsteady radius $a(t) = a_0\sqrt{1-\beta t}$, where a_0 is a constant and β is a constant of expansion/contraction strength. We also assume the temperature at the cylinder surface is varying as a function of time $T_w(z,t)$, and the ambient fluid temperature is T_∞, with $T_w > T_\infty$. In the absence of body force and base on the assumptions of axisymmetric flow, the Navier–Stokes and energy equations in the cylindrical coordinate are given as

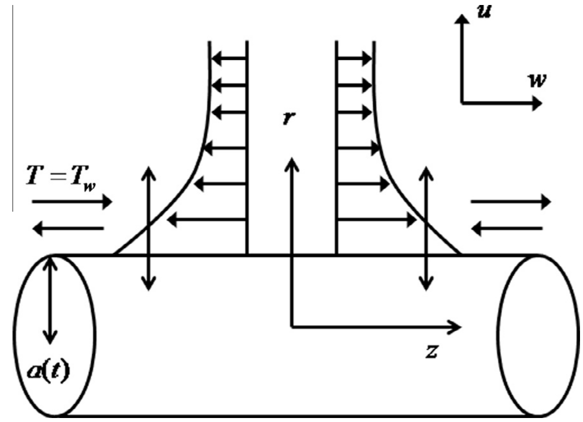

Figure 1 Physical model and coordinate system.

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right), \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right), \tag{3}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right), \tag{4}$$

where z and r are the cylindrical polar coordinates measured in axial and radial directions, u and w are the velocity components in the r and z directions, respectively, p is the pressure of the fluid, ρ is the fluid density, ν is the kinematic viscosity, c_p is specific heat, k is the thermal conductivity and T is temperature of the fluid.

The relevant boundary conditions for the present problem are

$$u = \frac{U}{\sqrt{1-\beta t}},\quad w = \varepsilon\frac{1}{a_0^2}\frac{4\nu z}{1-\beta t} + N\mu\frac{\partial w}{\partial r},\quad T = T_w(z,t) \text{ at } r = a(t),$$
$$w \to 0,\quad T \to T_\infty \text{ as } r \to \infty, \tag{5}$$

where $U(<0)$ is the constant mass transfer (suction) velocity, $\varepsilon = 1$ for stretching cylinder, $\varepsilon = -1$ for shrinking cylinder and $\varepsilon = 0$ for the static cylinder, μ is the dynamic viscosity, b is the constant, $N = N_1\sqrt{1-\beta t}$ is the velocity slip length which is changed with time and N_1 is the initial value of velocity slip parameter and has dimension $(\text{velocity})^{-1}$ and β has the dimension of $(\text{time})^{-1}$. The no-slip condition can be obtained for $N = 0$ in this case. The surface temperature $T_w(z,t)$ of the fluid is defined as

$$T_w(z,t) = T_\infty + \frac{bz}{a_0\nu(1-\beta t)}, \tag{6}$$

where $b(>0)$ is a constant.

The governing flow equations can be reduced into set of ordinary differential equations with the following similarity transformations