Ain Shams Engineering Journal (2015) 6, 967–975

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

ENGINEERING PHYSICS AND MATHEMATICS

Some analytical solutions for flows of Casson fluid with slip boundary conditions

K. Ramesh, M. Devakar *

Department of Mathematics, Visvesvaraya National Institute of Technology, Nagpur 440010, India

Received 23 July 2014; revised 29 January 2015; accepted 21 February 2015
Available online 14 April 2015

KEYWORDS

Casson fluid;
Slip boundary conditions;
Non-linear differential equations

Abstract In the present paper, we have studied three fundamental flows namely Couette, Poiseuille and generalized Couette flows of an incompressible Casson fluid between parallel plates using slip boundary conditions. The equations governing the flow of Casson fluid are non-linear in nature. Analytical solutions of the non-linear governing equations with non-linear boundary conditions are obtained for each case. The effect of the various parameters on the velocity and volume flow rate for each problem is studied and the results are presented through graphs. It is observed that, the presence of Casson number decreases the velocity and volume flow rate of the fluid. Increasing of slip parameter increases the velocity and volume flow rate in both Poiseuille and generalized Couette flows.

1. Introduction

The study of non-Newtonian fluids has gained great importance mainly due to their huge range of practical applications in engineering and industry. Numerous research workers have studied diverse flow problems related to several non-Newtonian fluids. Among the non-Newtonian fluids, Casson fluid has attracted more attention of researchers due to its applications in the fields of metallurgy, food processing, drilling operations and bioengineering operations. Some more applications can be seen in the manufacturing of pharmaceutical products, coal in water, china clay, paints, synthetic lubricants and biological fluids such as synovial fluids, sewage sludge, jelly, tomato sauce, honey, soup and blood due to its contents such as plasma, fibrinogen and protein [1]. Casson fluid is categorized as a non-Newtonian fluid due to its rheological characteristics. These characteristics show shear stress–strain relationships that are significantly different from Newtonian fluids. The success of theoretical and experimental investigations [2–5] have made Casson fluid model popular among non-Newtonian fluid models. In view of this, many researchers have studied the flow of Casson fluid in different geometries (see [6–19] and the references therein). The study of Couette, Poiseuille and generalized Couette flows of non-Newtonian fluids has attracted the attention of researchers in fluid dynamics due to their applications in engineering and industry. These flows also give the exact solutions of governing differential equations which may be used for comparing the results of the flow problems with complicated geometries.

* Corresponding author. +91 712 2801407; fax: +91 712 2223230.
E-mail addresses: ramesh.katta1@gmail.com (K. Ramesh), m_devakar@yahoo.co.in (M. Devakar).

Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2015.02.007
2090-4479 © 2015 Production and hosting by Elsevier B.V. on behalf of Ain Shams University.

These flows are unidirectional in nature and used in polymer processing such as extraction and die flow, injection molding, blow molding and asthenospheric flows [20–22].

A majority of Newtonian and non-Newtonian fluid flows have been examined taking no-slip condition into account and very little energy has been devoted to study the flows of non-Newtonian fluids with slip boundary conditions. The works of O'Neill et al. [23] and Basset [24] evidence the existence of slip at the solid boundary. For most of the problems, at the micro scale, the boundary surface roughness may be significant. If the boundary is smooth, the usual no-slip boundary condition might not hold and the fluid slippage might occur at the solid surface [25–28]. In fact, long back (1823) Navier [29] has proposed a slip boundary condition which suggests that the tangential velocity of the fluid relative to the solid boundary at a point on its surface is proportional to the tangential stress acting at that point. Neto et al. [30] have made a review of experimental studies on the boundary slip of Newtonian liquids. In view of this, various researchers used the slip boundary condition to study flow problems in different configurations. Hakeem et al. [31] have examined the slip effects on peristaltic transport of power law fluid through an inclined tube. Chen and Zhu [32] have obtained the analytical solutions of Couette-Poiseuille flow of Bingham fluids between two porous parallel plates with slip conditions. Tang [33] studied the analysis on creeping channel flows of compressible fluids subject to wall slip. Hayat et al. have analyzed the effect of slip condition on peristaltic motion of Phan–Thien–Tanner fluid [34] and the effect of the slip condition on the flows of an Oldroyd 6-constant fluid [35]. Ellahi et al. [36,37] have obtained the exact solutions of fundamental flows of third grade fluid and Oldroyd 8-constant fluid with non-linear slip conditions. Khaled and Vafai [38] have presented the exact solutions of Stokes and Couette flows due to an oscillating wall with slip boundary conditions. Tripathi et al. [39] have studied the influence of slip condition on the peristaltic transport of a viscoelastic fluid with fractional burger's model. Ferras et al. [40] have obtained analytical solutions for Newtonian and inelastic non-Newtonian flows with wall slip conditions. Kaoullas and Georgiou [41] have studied the Poiseuille flows of Newtonian fluid with slip and non-zero slip yield stress. Abelman et al. [42] have presented a numerical solution for the steady Couette flow of a thermodynamic compatible third grade fluid filling the porous space in a rotating frame with partial slip effects. Hron et al. [43] have investigated the influence of Navier's slip on the boundary for flows of incompressible fluids. Recently Devakar et al. [44] have studied Couette, Poiseuille and generalized Couette flows of couple stress fluid under slip conditions. Akbar et al. [45] have discussed the simulation of thermal and velocity slip on the peristaltic flow of a Johnson–Segalman fluid in an inclined asymmetric channel. Akbar and Nadeem [46] have presented the thermal and velocity slip effects on the peristaltic flow of a six constant Jeffrey's fluid model.

To the best of author's knowledge, no researcher has attempted to study the Couette, Poiseuille and generalized Couette flows of Casson fluid with slip boundary conditions. Hence, the aim of present paper is to find the analytical solutions of plane Couette, Poiseuille and generalized Couette flows under slip boundary conditions. The effect of the various flow parameters on each problem is studied and the results are presented through graphs. The paper is organized in terms of 7 sections. Section 2 describes basic equations governing the flows under consideration. The Couette, Poiseuille and generalized Couette flow problems have been solved respectively in Sections 3–5. The results are discussed in Section 6 and the conclusions of the study are presented in the last section.

2. Basic equations

The modified stress–strain relationship for the Casson fluid in tensor format is given by Mernone and Mazumdar [10] as

$$\sigma_{ij} = -p\delta_{ij} + 2\mu(J_2)V_{ij} \tag{1}$$

where

$$\mu(J_2) = \left[K_c + \left(\frac{\tau_0}{2}\right)^{\frac{1}{2}} J_2^{-\frac{1}{4}}\right]^2 = \mu(\text{say}) \tag{2}$$

Here σ_{ij} is the Cauchy stress tensor, p denotes the pressure, δ_{ij} is the Kronecker delta, μ is the apparent viscosity, τ_0 is the yield stress, $K_c = \eta^{\frac{1}{2}}$ (η is the Casson's coefficient of viscosity), V_{ij} and J_2 are known as deformation tensor and second invariant of a tensor respectively and these are defined as

$$V_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad \text{and}$$

$$J_2 = \frac{1}{2}V_{ij}V_{ij} = \frac{1}{2}\left(V_{11}^2 + V_{22}^2 + V_{33}^2 + 2V_{12}^2 + 2V_{13}^2 + 2V_{23}^2\right)$$

with

$$V_{11} = \frac{\partial u_1}{\partial x_1}, \quad V_{22} = \frac{\partial u_2}{\partial x_2}, \quad V_{33} = \frac{\partial u_3}{\partial x_3}, \quad V_{12} = V_{21} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right),$$
$$V_{13} = V_{31} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right), \quad V_{23} = V_{32} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \tag{3}$$

where $\bar{q} = (u_1, u_2, u_3)$ is the velocity vector.

With these, the equation governing the flow of an incompressible Casson fluid are,

$$\text{div } \bar{q} = 0 \tag{4}$$

and

$$\rho\frac{Du_i}{Dt} = \frac{\partial}{\partial x_j}\sigma_{ij} \tag{5}$$

where ρ is the density of the fluid and $\frac{D}{Dt}$ is the material derivative. For unidirectional steady flow between parallel plates, taking $(x_1, x_2, x_3) = (x, y, z)$ and $(u_1, u_2, u_3) = (u, v, w)$, the velocity field $\bar{q} = (u(y), 0, 0)$. This velocity automatically satisfies the continuity Eq. (4) and reduces the momentum Eq. (5) as,

$$\frac{d}{dy}\left[\left(K_c + \left(\frac{\tau_0}{\left(\frac{du}{dy}\right)}\right)^{\frac{1}{2}}\right)^2 \frac{du}{dy}\right] - \frac{dp}{dx} = 0 \tag{6}$$

3. Plane Couette flow

An incompressible Casson fluid is bounded between two infinitely long horizontal rigid parallel plates distant $2h$ apart. The pressure gradient between parallel plates is assumed to