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Dynamic behavior of a rotating delaminated composite beam including rotary inertia and shear deformation effects

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Abstract A finite element (FE) model is developed to study the free vibration of a rotating laminated composite beam with a single delamination. The rotary inertia and shear deformation effects, as well as the bending–extension, bending–twist and extension–twist coupling terms are taken into account in the FE model. Comparison between the numerical results of the present model and the results published in the literature verifies the validity of the present model. Furthermore, the effects of various parameters, such as delamination size and location, fiber orientation, hub radius, material anisotropy and rotating speed, on the vibration of the beam are studied in detail. These results provide useful information in the study of the free vibration of rotating delaminated composite beams.

1. Introduction

The rotating beams made of composite materials are widely used for various engineering applications, such as robotic manipulators, wind turbines, helicopter blades and aircraft propellers. The improved mechanical properties of laminated composites, such as strength-to-weight and stiffness-to-weight ratios, in comparison with the conventional metallic materials are some of the factors that have contributed to its advancement. However, composites are subjected to a wide range of damages induced during their fabrication or service life, which may significantly reduce their structural performance. One of the commonly encountered types of defects or damages in the multi-layered composite structures is delamination.

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Nomenclature

L	beam length
$b \times h$	rectangular cross-section of the beam
L_1	delamination lengthwise location
L_2	delamination length
r_0	offset distance
M	end mass
J	moment of inertia of the end mass
Ω	rotating speed
h	beam thickness
h_i	thickness of the ith sub-beam
T	kinetic energy
U	potential energy
ρ_i	ith layer density
(u, w)	the LCB mid-plane displacements in the $\hat{x}$ and $\hat{y}$ directions, respectively
$(\psi_{\hat{x}}, \psi_{\hat{y}})$	the mid-plane bending slopes
$N_{\hat{x}}$	in-plane force
$(M_{\hat{x}}, M_{\hat{x}\hat{y}})$	bending and twisting moments
$Q_{\hat{x}\hat{z}}$	resultant shear force
$\varepsilon^0_{\hat{x}}$	mid-plane strain
$(\kappa_{\hat{x}}, \kappa_{\hat{x}\hat{y}})$	bending and twisting curvatures
$\varepsilon_{\hat{x}\hat{z}}$	shear strain
$\overline{A}_{ij}$	modified extensional stiffness
$\overline{B}_{ij}$	modified bending–extension coupling stiffness
$\overline{D}_{ij}$	modified bending stiffness
A_{ij}	transverse shear stiffness
F_i	centrifugal force
ω	circular frequency
k_s	shear correction factor
(A, I)	area and second moment of area, respectively
R	radius of gyration
ω_s	natural frequency of the non-rotating intact beam
E	Young Modulus
$(\eta' = \Omega L^2\sqrt{\rho A/EI}, \eta = \Omega L^2\sqrt{\rho/(E_{11}h^2)})$	dimensionless rotating speed
$(\bar{\omega}' = \omega L^2\sqrt{\rho A/EI}, \bar{\omega} = \omega L^2\sqrt{\rho/E_{11}h^2})$	dimensionless natural frequency
(e_2, e_3)	distances between the neutral axis of the sub-beams 2 and 3 with the neutral axis of the intact part, respectively
$(r = R/L, \alpha = r_0/L, \overline{L}_2 = L_2/L, \mu = M/\rho bhL, \sigma = J/\rho bhL^3)$	non-dimensional parameters used in the numerical analysis

Depending on its application, composite structures may require the use of angle ply and unsymmetrical laminates. Thus, bending–extension, bending–twist, and extension–twist coupling terms need to be considered in the analysis of composites [1–5]. Furthermore, the Poisson's effect, which is often neglected in one-dimensional laminated beam analysis, has been found to be significant in the analysis of composite beams with angle-plies and unsymmetrical layups [1–5].

Free vibration analysis of the intact rotating beams and delaminated non-rotating beams based on the classical beam theory (CBT) and the Timoshenko beam theory has received a good amount of attention in the literature [6–22]. The free vibrations of an isotropic beam with a through-width delamination by using four Euler–Bernoulli sub-beams connected at the delamination boundaries were studied by Wang et al. [6]. According to this study interpenetration of the delaminated sub-beams was seen, which was physically impossible to occur in the case of off-mid-plane delaminations. This is because the delaminated regions were assumed to deform "freely" without touching each other (free mode) and thus have different transverse deformations. To avoid this kind of incompatibility, Mujumdar and Suryanarayan [7] proposed a model in which they assumed that the delaminated layers are constrained to have identical transverse deformations, which was called the constrained mode in contrast with the free mode by Wang et al. [6]. However, the constrained mode model failed to predict the opening in the mode shapes found in the experiments by Shen and Grady [8]. Analytical and numerical solutions for beams with single and multiple delaminations have been presented by many researchers based on the CBT [9–14] and Timoshenko beam theory [15–17]. These studies emphasize on the influence of the free and constrained modes between the delaminated layers. It should be noted that the Timoshenko beam theory takes into account the shear deformation and rotary inertia effects, which are ignored in the CBT, making it suitable to study the static and dynamic behavior of thick beams. Moreover, in this case, the equation of motion is complex, and even obtaining an approximate solution is much more difficult than the CBT.

The free vibration of rotating intact beam has received considerable attention from researchers [18–22]. Kuo et al. [18] have investigated the influence of taper ratio, elastic root restraints, tip mass and rotating speed on the vibration of rotating non-uniform beams based on the CBT. Natural frequencies of rotating tapered Timoshenko beam have been presented for different combinations of the fixed, hinged and free end conditions by means of a new tapered finite beam element that accounts for the effect of shear deformation, rotary inertia, and the centrifugal stiffening due to beam rotation [19]. Du et al. [20] have presented a convergent power series expression to solve analytically for the exact natural frequencies and mode shapes of rotating Timoshenko beams. The effects of angular velocity, shear deformation and rotary inertia on the dynamic behavior of rotating beams have been evaluated. The free vibration analysis of rotating tapered beams has been investigated using the dynamic stiffness method by Su et al. [21]. The range of considered problems included beams for which the depth and/or width of the cross-section vary linearly along the length. Das et al. [22] have presented analytical solution for the free vibration of a rotating beam with nonlinear spring–mass system. The solution has been obtained by applying the method of multiple timescale directly to the nonlinear partial differential equations and the boundary conditions.

Dynamic analysis of the rotating delaminated beam has received limited attention. Recently, Liu and Shu [23] presented analytical solutions for the free vibrations of rotating isotropic beams with multiple delaminations. The Timoshenko beam theory and both the free mode and the