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On Cattaneo–Christov heat flux in MHD flow of Oldroyd-B fluid with homogeneous–heterogeneous reactions

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ABSTRACT

This paper investigates the steady two-dimensional magnetohydrodynamic (MHD) flow of an Oldroyd-B fluid over a stretching surface with homogeneous–heterogeneous reactions. Characteristics of relaxation time for heat flux are captured by employing new heat flux model proposed by Christov. A system of ordinary differential equations is obtained by using suitable transformations. Convergent series solutions are derived. Impacts of various pertinent parameters on the velocity, temperature and concentration are discussed. Analysis of the obtained results shows that fluid relaxation and retardation time constants have reverse behavior on the velocity and concentration fields. Also temperature distribution decreases for larger values of thermal relaxation time.

1. Introduction

The non-Newtonian fluids at present have wide applications in industry and engineering. Some of their common examples are polymer solutions, paints, certain oils, exotic lubricants, colloidal and suspension solutions, clay coatings and cosmetic products. Due to diverse physical structures of these fluids, there is not even a single constitutive relationship which can predict all the salient features of non-Newtonian fluids. Generally these fluids have been classified into three main categories: (i) the differential type, (ii) the rate type and (iii) the integral type. Numerous attempts have been made in the past for the flows of differential type fluids. This is because of the fact that constitutive equations in the differential type fluids are much easier and one can explicitly obtain the shear stresses in terms of velocity components. Existing literature indicates that little attention has been given to the flows of rate type material. The simplest subclass of rate type fluid is Maxwell fluid. This fluid model can only describe relaxation time but it provides no information about its retardation time. On the other hand an Oldroyd-B fluid [1–10] has a measurable relaxation and retardation times and it can capture the viscoelastic features of dilute polymeric solutions under general flow conditions.

The heat flux model proposed by Fourier [11] is the most successful relation for the description of heat transfer mechanism in various situations. However it has a major limitation that it yields a parabolic energy equation which indicates that initial disturbance is instantly experienced by the medium under consideration. This feature is referred in literature as “Paradox of heat conduction”. To overcome this situation, various researchers have proposed modifications in the Fourier's heat conduction law. Cattaneo [12] suggested a modification of Fourier's model by incorporating relaxation time for heat flux. It is seen that such consideration produces hyperbolic energy equation and it allows the transportation of heat through the propagation of thermal waves with finite speed. Such heat transportation process has exciting practical applications that span from nanofluid flows to the modeling of skin burn injury (see Tibullo and Zampoli [13] and references therein). Christov [14] further modified the time derivative in Maxwell–Cattaneo's model with Oldroyd's upper-convected derivative in order to preserve the material-invariant formulation. This modification in literature is recognized as Cattaneo–Christov heat flux model. Ciarletta and Straughan [15] proved the uniqueness of the solutions for the Cattaneo–Christov equations. Straughan [16] examined the natural convection in horizontal layer of an incompressible Newtonian fluid. Han et al. [17] studied the slip flow and heat transfer in flow of Maxwell fluid subject to Cattaneo–Christov model. They solved the governing problem analytically by homotopy analysis method (HAM). Mustafa [18] examined Cattaneo–Christov heat flux in rotating flow of upper-convected Maxwell fluid.

Many chemically reacting systems involve both homogeneous

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and heterogeneous reactions. Some of the reactions have the ability to proceed very slowly or not at all, except in the presence of a catalyst. The interaction between the homogeneous and heterogeneous reactions is very complex involving the production and consumption of reactant species at different rates both within the fluid and on the catalytic surfaces such as reactions occurring in food processing, hydrometallurgical industry, manufacturing of ceramics and polymer production, fog formation and dispersion, chemical processing equipment design, crops damage via freezing, cooling towers and temperature distribution and moisture over agricultural fields and groves of fruit trees. A model for isothermal homogeneous–heterogeneous reactions in boundary layer flow of viscous fluid past a flat plate is studied by Merkin [19]. He presented the homogeneous reaction by cubic autocatalysis and the heterogeneous reaction with a first order process. It is shown that the surface reaction is the dominant mechanism near the leading edge of plate. Chaudhary and Merkin [20] studied homogenous–heterogeneous reactions in boundary layer flow of viscous fluid. They computed numerical solution near the leading edge of a flat plate. Bachok et al. [21] analyzed stagnation-point flow towards a stretching sheet with homogeneous–heterogeneous reactions. Khan and Pop [22] investigated effects of homogeneous–heterogeneous reactions in the flow of viscoelastic fluid towards a stretching sheet. Shaw et al. [23] examined homogeneous–heterogeneous reactions in micropolar fluid flow from a permeable stretching or shrinking sheet in a porous medium. Kameswaran et al. [24] extended the work of Khan and Pop [22] for viscous nanofluid over a porous stretching sheet. Hayat et al. [25] analyzed homogeneous–heterogeneous reactions in the stagnation point flow of carbon nanotubes with Newtonian heating. Effect of homogeneous–heterogeneous reactions in flow of Powell–Eyring fluid is examined by Hayat et al. [26].

Many analytical methods like differential transformation method (DTM) [27–29], least square method (LSM) [30,31], homotopy analysis method (HAM) [32–40], etc. are observed in the literature for solving the physical and engineering problems. Homotopy analysis method (HAM) is one of the most efficient methods in solving different type of nonlinear equations such as coupled, decoupled, homogeneous and non-homogeneous. Many previous analytic methods have some restrictions in dealing with nonlinear equations. Unlike perturbation method, HAM is independent of any small or large parameters. Also HAM provides us with great freedom to choose initial guesses and auxiliary parameters to control and adjust the convergence region which is a main lack of other several techniques.

Motivated by such facts, the purpose of this paper is to study the heat and mass transfer in MHD flow of an Oldroyd-B fluid over a stretching sheet with Cattaneo–Christov heat flux. Influence of homogeneous–heterogeneous reactions is also examined. Convergent solutions are obtained by homotopy analysis method (HAM). The behaviors of different parameters on the physical quantities of interest have been examined graphically.

2. Model development

Consider the steady two-dimensional incompressible flow of an Oldroyd-B fluid bounded by a linear stretching sheet. The velocity of sheet is assumed $u_w = cx$ where $c > 0$ is the stretching rate. The sheet is kept at constant temperature T_w whereas T_∞ being the ambient temperature such that $T_w > T_\infty$. A uniform magnetic field of strength B_0 is applied in y-direction. Electric and induced magnetic fields are neglected. Flow analysis is carried out with homogeneous–heterogeneous reactions. The homogeneous reaction for cubic autocatalysis can be expressed as follows:

$$A + 2B \rightarrow 3B \quad \text{rate} = k_c a b^2, \tag{1}$$

while first-order isothermal reaction on the catalyst surface is presented in the form

$$A \rightarrow B \quad \text{rate} = k_s a, \tag{2}$$

where a and b are the concentrations of the chemical species A and B and k_c and k_s are the rate constants. We assume that both reaction processes are isothermal. Under these assumptions, the boundary layer equations governing the flow can be expressed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \Lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = \nu\frac{\partial^2 u}{\partial y^2} - \nu\Lambda_2\left(u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) - \sigma B_0^2\left(u + \Lambda_1 v\frac{\partial u}{\partial y}\right), \tag{4}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = -\nabla.\mathbf{q}, \tag{5}$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\frac{\partial^2 a}{\partial y^2} - k_c a b^2, \tag{6}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\frac{\partial^2 b}{\partial y^2} + k_c a b^2, \tag{7}$$

The corresponding boundary conditions are

$$u = u_w, \quad v = 0, \quad T = T_w, \quad D_A\frac{\partial a}{\partial y} = k_s a, \quad D_B\frac{\partial b}{\partial y} = -k_s a \quad \text{at } y = 0,$$

$$u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad a \rightarrow a_0, \quad b \rightarrow 0 \quad \text{as } y \rightarrow \infty, \tag{8}$$

where u and v are the velocity components in the x- and y-directions respectively, ν the kinematic viscosity of fluid, ρ the fluid density, Λ_1 the relaxation time, Λ_2 the retardation time, σ the electrical conductivity, C_p the specific heat, D_A and D_B the respective diffusion species coefficients of A and B, a_0 the positive dimensional constant and the heat flux $\mathbf{q}$ satisfies the following relationship [13]:

$$\mathbf{q} + \lambda\left(\frac{\partial \mathbf{q}}{\partial t} + \mathbf{V}.\nabla\mathbf{q} - \mathbf{q}.\nabla\mathbf{V} + (\nabla.\mathbf{V})\mathbf{q}\right) = -k\nabla T, \tag{9}$$

in which λ is the relaxation time of heat flux and k the thermal conductivity of fluid. Note that Eq. (9) simplified to Fourier's law for λ=0. Since the fluid is incompressible so $\nabla.\mathbf{V} = 0$ and Eq. (9) becomes

$$\mathbf{q} + \lambda\left(\frac{\partial \mathbf{q}}{\partial t} + \mathbf{V}.\nabla\mathbf{q} - \mathbf{q}.\nabla\mathbf{V}\right) = -k\nabla T. \tag{10}$$

The above equation is taken in absence of magnetohydrodynamics for simplicity. Eliminating $\mathbf{q}$ between Eqs. (5) and (10), we obtain following governing equation [14]: