Journal of Magnetism and Magnetic Materials 401 (2016) 354–361

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Influence of inclined Lorentz forces on boundary layer flow of Casson fluid over an impermeable stretching sheet with heat transfer

A.K. Abdul Hakeem [a,*], P. Renuka [b], N. Vishnu Ganesh [a], R. Kalaivanan [a], B. Ganga [c]

[a] Department of Mathematics, Sri Ramakrishna Mission Vidyalaya, College of Arts and Science, Coimbatore 641 020, India
[b] Department of Mathematics, Erode Sengunthar Engineering college, Erode 638 057, India
[c] Department of Mathematics, Providence College for Women, Coonoor 643 104, India

ARTICLE INFO

Article history:
Received 27 July 2015
Received in revised form
1 October 2015
Accepted 9 October 2015

Keywords:
Boundary layer flow
Casson fluid
Inclined magnetic field
Lorentz force
Thermal radiation

ABSTRACT

The inclined magnetic field effect on the boundary layer flow of a Casson model non-Newtonian fluid over a stretching sheet in the existence of thermal radiation and velocity slip boundary condition is investigated for both prescribed surface temperature and power law of surface heat flux cases. It is assumed that the magnetic field is applied with an aligned angle which varied from 0° to 90°. Both analytical and numerical solutions are obtained for the transformed non-dimensional ODE's using confluent hypergeometric function and fourth order Runge–Kutta method with shooting technique respectively. The combined effects of inclined magnetic field with other pertinent parameters such as Casson parameter, velocity slip parameter, radiation parameter and Prandtl number on velocity profile, temperature profile, local skin friction coefficient, local Nusselt number and non-dimensional wall temperature are discussed through graphs. It is found that the aligned angle plays a vital role in controlling the magnetic field strength on the Casson fluid flow region and the increasing values of aligned angle of the magnetic field lead to decrease the skin friction coefficient and the Nusselt number and increase the non-dimensional wall temperature.

1. Introduction

Magnetohydrodynamics (MHD) is the study of the interaction of conducting fluids with electromagnetic phenomena which has important applications in industrial fields. For example, equipments such as MHD generators, flow meters, bearings, pumps and boundary layer control are affected by the interaction between the electrically conducting fluid and a magnetic field. Hydromagnetic boundary layers are observed in several technical systems employing liquid metal and plasma flow transverse of magnetic fields. In these cases, the flow control can be realized by the Lorentz force. Recently, many publications discussed the effect of magnetic field on the boundary layer flow problems along a stretching sheet in transverse direction [1–7]. In most of the publications the no-slip boundary condition is considered, but in some situations such as suspensions, emulsions, polymer solution and foams, the no-slip condition is inadequate. Keeping this in view, the following researchers have investigated the boundary layer flow problems over a stretching sheet in the presence of slip boundary condition and transverse magnetic field [8–14].

Casson fluid model is one of the non-Newtonian fluid model. This was first introduced by Casson in 1995. Casson model is based on a structure model of the interactive behavior of solid and liquid phases of a two-phase suspension. Casson fluid exhibits yield stress. It is well-known that Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear, i.e., if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move. The examples of Casson fluid are of the type as follows: honey, soup, jelly, tomato sauce and concentrated fruit juices. Human blood can also be treated as Casson fluid. Many researchers have been attracted by Casson fluid model and discussed the boundary layer flow problem of Casson fluid with various physical effects such as magnetic field and slip conditions [15–20]. All these studies on Casson fluid have reported the boundary layer flow over stretching sheet in the absence of aligned magnetic field.

Keeping this in mind, in the present paper we have studied the influences of inclined magnetic field on the boundary layer flow of a Casson fluid over a stretching sheet in the presence of thermal radiation and velocity slip boundary condition for both PST and PHF cases. Similarity transformations are applied to convert the governing nonlinear partial differential equations into nonlinear

* Corresponding author.
E-mail address: abdulhakeem6@gmail.com (A.K. Abdul Hakeem).

http://dx.doi.org/10.1016/j.jmmm.2015.10.026
0304-8853/

Table 1
Comparison of $-\theta_\eta(0)$ and $g(0)$ in PST and PHF cases.

Mn	Pr	Turkyilmazoglu [8]		Present results			
				Analytical		Numerical	
		$-\theta_\eta(0)$	$g(0)$	$-\theta_\eta(0)$	$g(0)$	$-\theta_\eta(0)$	$g(0)$
0	1	1.33333	0.75000	1.33333	0.75000	1.3333333	0.7500000
	5	3.31648	–	3.31648	0.30152	3.3164824	0.3015242
1	1	1.21577	0.82252	1.21577	0.82252	1.2157726	0.8225221
	5	–	–	3.20720	0.31179	3.2072054	0.3117979

ordinary differential equations which are then solved analytically using confluent hypergeometric function and numerically by the fourth order Runge–Kutta method with shooting technique.

2. Mathematical formulation

Consider a steady, laminar, two-dimensional boundary layer flow of an incompressible, Casson fluid over a stretching sheet. Aligned magnetic field of strength B_0 is applied along y direction, with an acute angle γ. At $\gamma=90°$ this magnetic field acts like transverse magnetic field (because $\sin(90°)=1$). It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field. Let x-axis be along the surface, y-axis being normal to it.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is [17]

$$\tau_{ij} = \begin{cases} 2(\mu_B + p_y/\sqrt{2\pi})e_{ij}, & \pi > \pi_c \\ 2(\mu_B + p_y/\sqrt{2\pi_c})e_{ij}, & \pi < \pi_c \end{cases}$$

Here, $\pi = e_{ij}e_{ij}$ and e_{ij} are the (i,j)th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid and p_y is the yield stress of the fluid. The equations governing the problem under consideration are given by

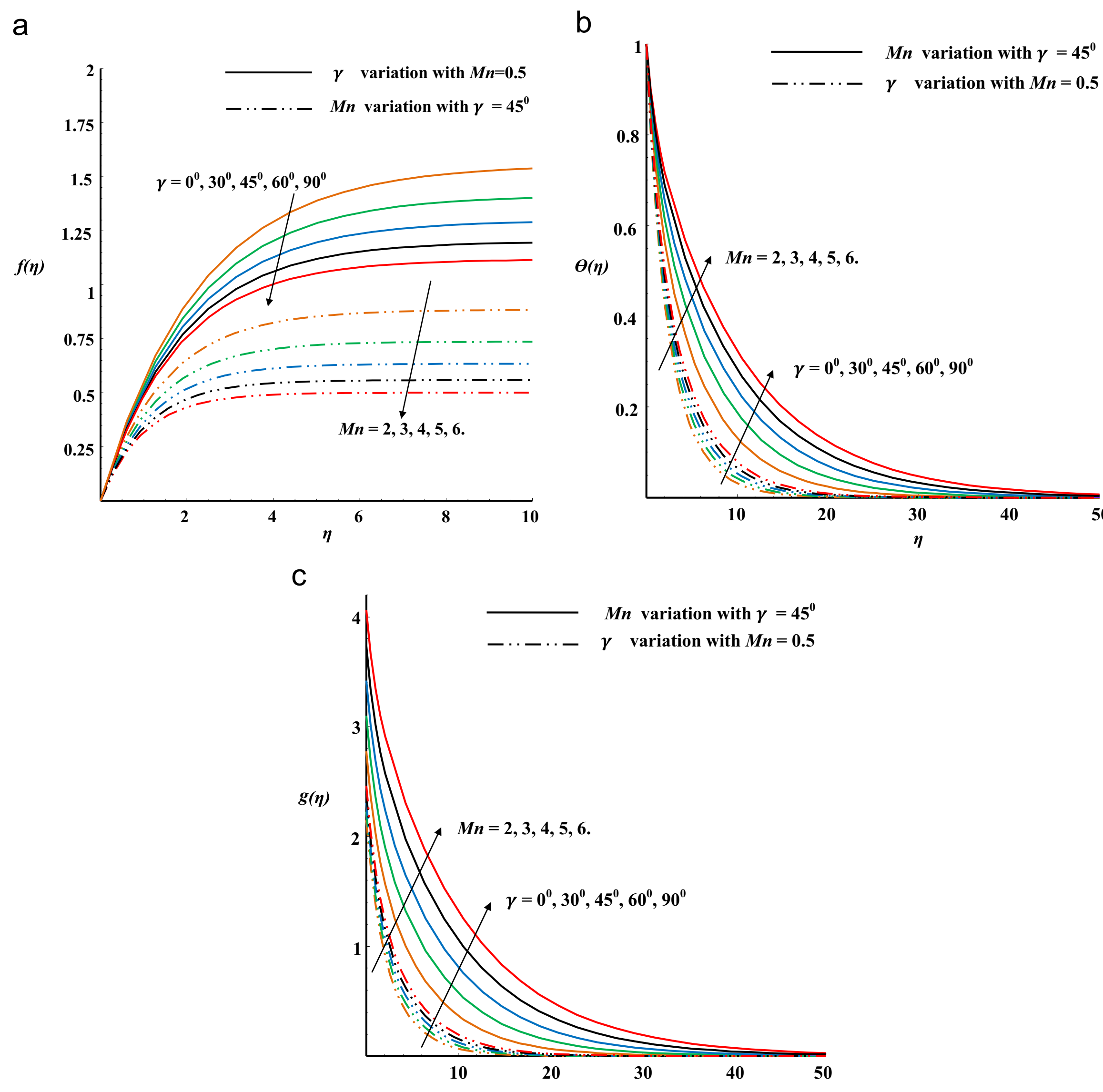

Fig. 1. (a) Effect of magnetic and angle parameters on velocity profile $f(\eta)$ with $L=1$ and $\beta=0.4$. (b) Effect of magnetic and angle parameters on temperature profile $\theta(\eta)$ (*PST case*) with $L=1$, $\beta=0.4$, $N=0.5$ and $Pr=0.71$ and (c) Effect of magnetic and angle parameters on temperature profile $g(\eta)$ (*PHF case*) with $L=1$, $\beta=0.4$, $N=0.5$ and $Pr=0.71$.