Ain Shams Engineering Journal (2015) 6, 1093–1104

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

ENGINEERING PHYSICS AND MATHEMATICS

Unsteady polar fluid model of blood flow through tapered ω-shape stenosed artery: Effects of catheter and velocity slip

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Received 10 August 2014; revised 11 December 2014; accepted 7 January 2015
Available online 24 March 2015

KEYWORDS

Couple stress fluid;
Blood flow;
Tapered artery;
Catheter;
Slip velocity

Abstract In this paper, we considered the pulsatile flow of blood through catheterized tapered artery in the presence of an ω-shaped stenosis. Blood flow is modelled as homogeneous incompressible couple stress fluid. Further the effects of velocity slip at the arterial wall are also examined. The analysis is carried out analytically and closed form solutions are obtained with the assumption of mild stenosis. In the present study, we analyze the effects of various fluid and geometric parameters on the physiological parameters such as resistance to flow and shear stress at the wall. The variation in the resistance to the flow and wall shear stress with respect to stenosis size (ϵ, ψ), radius of the catheter (R_c), couple stress fluid parameters (β, ω), Reynolds number (Re) and pulsatile parameter (σ) has been studied. In particular shear stress at the wall is reckoned at both the locations corresponding to the maximum height of the stenosis. It has been observed that this physiological parameter is independent of the location of the maximum height in case of nontapered artery while these locations significantly impact the shear stress at the wall in case of tapered artery. The locations of the critical and maximum heights with corresponding annular radii are summarized in the form of Table 1. We also focussed our attention on the analysis of the wall shear stress over the entire stenosis region for various values of the geometric and fluid parameters. It is observed that the impedance and wall shear stress are increasing with increase in the radius of catheter and stenosis size while they are decreasing as the tapered parameter and the couple stress fluid parameters are increasing. It is observed that slip velocity and diverging tapered artery facilitate the fluid flow.

1. Introduction

Cardiovascular diseases (CVDs) are the group of disorders of heart and blood vessels. CVDs include coronary heart disease, cerebrovascular disease, peripheral arterial disease, rheumatic heart disease, congenital heart disease and deep vein

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Peer review under responsibility of Ain Shams University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2015.01.003
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thrombosis and pulmonary embolism. Over the period of past few decades it has been the leading cause of death worldwide [1]. Lot of research efforts are going on to prevent, control and cure these disorders. Most of the deaths occur because of heart attacks and strokes apart from conditions of ischaemia, atherosclerosis and thrombosis. Heart attacks and strokes are usually acute events mainly caused by blockage/s that prevent the flowing of blood to the heart or brain. The build-up of fatty deposits on the inner walls of the blood vessels, that supply blood to the heart or brain is one of the most-common causes. Strokes can be caused by bleeding from a blood vessel in the brain or from blood clots. The abnormal narrowing of blood vessels in various locations of cardiovascular system due to the deposition of the cholesterol and other fatty substances leads to a medical condition called as stenosis [2]. Stenoses lead to circulatory disorders such as atherosclerosis. The ethological studies on stenosis suggest that deposition of calcium, fatty components and cholesterol on the inner walls of the artery prevent the flowing of blood leading to rupture of the artery and thrombosis. Thrombus can form emboli which occlude the smaller vessels.

The dynamics of the blood flow is drastically affected in all these conditions and has adverse effects on the blood circulation and its control by cardiovascular system. Stenosis increases the resistance to the flow of blood in arteries resulting in increased blood pressure. It is now a well established fact that stenosis induces substantial changes in blood flow velocity, pressure distribution, wall shear stress and impedance. Currently most of these conditions cannot be detected in routine check-ups and CT scan remains a major technique to diagnose stenosis conditions. It is imperative to understand the behaviour of blood flow in a stenosed artery which is quite different in comparison to the normal ones. Information about the flow parameters such as velocity, flow rate, pressure drop in diseased vessel can be crucial and can help to save the further fatality and prevent the occurrence of the disease. It can help patients to get right treatment at right time. The mathematical modelling and numerical simulation studies have the huge potential and can very well interpret existing in vivo data and eventually help in the improved diagnosis. The assumptions of blood flow parameters made from mathematical modelling can be crucial and life saving. In this paper we try to model the blood flow in stenosed artery to understand the behaviour of blood and connect it to the possible leading medical condition.

Initial understanding of blood flow dynamics was done by considering blood as a Newtonian fluid [3–5]. But theoretical and experimental investigations indicated that blood cannot be treated as a single phase homogeneous viscous fluid while flowing through small arteries [6,7]. It is now well established that blood is a suspension of corpuscles (cellular particles) in an aqueous saline solution of plasma which indicates that blood is having a non-Newtonian structure. Siddiqui et al. [8] have investigated the pulsatile nature of blood by modelling blood as a Casson fluid. They observed that the yield stress increases with a decrease in the mean and steady flow rates. Sankar et al. [9] observed the effects of non-symmetric stenosis on the physiological properties of the flow by treating blood as Herschel Bulkley fluid. Varshney et al. [10] noticed the effect of time-dependent radius of the artery on flow rate, wall shear stress, etc., by considering the blood as power-law fluid. The influence of heat and mass transfer on blood flow through asymmetric stenosis was discussed by Akbar et al. [11,12], where in blood is modelled as Jefferey fluid and Sisko fluid respectively. A perturbation method was used to examine biomechanical analysis of Prandtl fluid flow through stenosed tapered artery by Akbar et al. [13].

Chakravarty et al. [14] have studied about the two dimensional blood flow through tapered arteries under stenotic condition. They mentioned that in most of the earlier studies, flow in the arteries has been considered in cylindrical tubes with uniform cross section. But in reality bifurcation of blood vessels at frequent intervals and variation in the vessel diameter with distance is well known and the most of the vessels could be considered as long, narrow and slowly tapering cones. Noreen et al. [15] have analysed the effects of vessel tapering together with the asymmetric stenosed tapered artery on the flow characteristics by considering blood as a Nano fluid. Here authors concluded that velocity profile is rising with the increase in slip velocity. Peristaltic Newtonian fluid of chyme flow through small intestine was modelled by Akbar et al. [16].

There are many treatments available for diagnosing and treating constricted vessels. Catheterization (thin, flexible tube) is one of them, in which balloon angioplasty is a specialized form of catheterization. These procedures are widely used in the medical field for treating the atherosclerosis. Insertion of the catheter in a tube creates an annular region between inner wall of the artery and outer wall of the catheter which influences the flow field such as pressure distribution, shear stress at the wall, resistance to flow (impedance). In view of its immense importance, the effect of the catheter on physiological parameters was discussed by the researchers [17,18]. In particular they considered pulsatile nature of blood flow when blood is modelled as non-Newtonian fluid.

The shapes of the stenosis in the above aforesaid studies have been considered to be radially symmetric or asymmetric. But while stenosis is maturing it may grow up in series manner, overlap with each other and it would be appear like ω-shape. Daniel et al. [19] observed the pressure gradient force, flow velocity, impedance and wall shear stress in the overlapping stenotic zone at the critical height and at the throats of such stenosis. Here they considered steady nature of flow. Srivastava et al. [20] explored the arterial blood flow through an overlapping stenosis by treating the blood as a Casson fluid. They figured impedance and shear stress for different stenosis heights. Chakravarty et al. [21] discussed the effects of the overlapping stenosis under low shear rate flow.

The presence of red cell slip at the vessel wall was recommended theoretically by Vand [22], experimentally by Bunnett [23] and Nubar [24], Chaturani et al. [25], etc. They used slip velocity at the wall in their analysis. Lately, Ponalagusamy [26], Biswas et al. [27] and Ponalagusamy [28] have developed mathematical models for blood flow through stenosed arterial segment, by taking a velocity slip condition at the constricted wall. The extensive discussion on symmetric and asymmetric slip velocity at the wall was done by Ghosh et al. [29]. Thus, it is very appropriate to consider velocity slip at the wall of the stenosed artery in blood flow modeling.

In the stenosed condition, substantial reduction in the lumen of an artery results in size effects (ratio of haematocrit to vessel diameter), which influences flow characteristics significantly. To study the size effect in the fluid flow, Stokes [30], Eringen [31] and Cowin [32] have proposed continuum