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Viscous potential flow analysis of magnetohydrodynamic capillary instability with heat and mass transfer

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Abstract A linear analysis of capillary instability of a cylindrical interface in the presence of axial magnetic field has been carried out when there is heat and mass transfer across the interface. Both fluids are taken as incompressible, viscous and magnetic with different kinematic viscosities and different magnetic permeabilities. Viscous potential flow theory is used for the investigation and a dispersion relation that accounts for the growth of axisymmetric waves is derived. Stability criterion is given by critical value of applied magnetic field as well as critical wave number and stability is discussed theoretically as well as numerically. Various graphs are drawn showing the effect of various physical parameters such as magnetic field strength, heat transfer capillary number, and permeability ratio, on the stability of the system. It has been observed that the axial magnetic field and heat and mass transfer both have stabilizing effect on the stability of the system.

1. Introduction

Capillary instability arises when a liquid cylinder in an infinite fluid collapses under the action of capillary forces due to surface tension [1,2]. The capillary instability occurs in various situations such as film boiling, Breaking of liquid jet and in many Chemical and Metallurgical processes. Plateau [3] studied the axisymmetric disturbances of capillary instability and observed that a long cylinder of liquid is unstable to the axisymmetric disturbances with wavelengths greater than $2\pi R$. Rayleigh [4] considered the stability of viscous liquid under capillary forces but he neglected the effect of surrounding fluid. Weber [5]

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studied the stability of a laminar jet considering the effect of viscosity and the surrounding air and concluded that the viscosity does not change the cut-off wave number predicated by the inviscid theory. Tomotica [6] considered the stability of a long cylindrical column of viscous liquid in the viscous fluid.

Magnetohydrodynamics is concerned with the ways in which magnetic fields can affect the fluid behavior. The stability of liquid jet of finite electrical conductivity in uniform axial magnetic field was studied by Chandrasekhar [1] for axisymmetric disturbances. He found that in the limit of infinite conductivity, the magnetic field has the effect of increasing the wavelength at which the capillary instability occurs. Taktarov [7] analyzed the stability of a magnetic fluid jet in the presence of a uniform magnetic fluid acting along the axis of the jet and observed that as the magnetic field is increased, the instability region shifts toward the long waves. Radwan [8] studied the effect of magnetic field on the capillary stability of inviscid fluids and observed that the magnetic field has stabilizing effect on the capillary instability. The instability of a compressible annular cylindrical fluid jet coaxial with a very dense fluid cylinder of negligible inertia endowed with surface tension and acting upon the electromagnetic force has been investigated by Radwan [9].

The study of heat and mass transfer across the interface is very important in many situations such as boiling heat transfer in chemical engineering and in geophysical problems. Hsieh [10,11] has established the general formulation of Rayleigh–Taylor instability and Kelvin–Helmholtz instability with heat and mass transfer across the liquid vapor interface. Hsieh [10] found that when the vapor layer is hotter than the liquid layer, the effect of heat and mass transfer tends to inhibit the growth of instability. Nayak and Chakraborty [12] formulated the Kelvin–Helmholtz instability of the cylindrical interface between the liquid and vapor phases with heat and mass transfer.

Viscous potential flow theory has played an important role in studying various stability problems. Tangential stresses are not considered in viscous potential theory and viscosity enters through normal stress balance [13]. Funada and Joseph [14] studied the viscous potential flow analysis of capillary instability and observed that viscous potential flow is better approximation of the exact solution than the inviscid model. Funada and Joseph [15] extended their work of capillary instability for viscoelastic fluids and observed that the growth rates are larger for viscoelastic fluids than for the equivalent Newtonian fluids. The viscous potential flow analysis of Kelvin–Helmholtz instability with heat and mass transfer in plane geometry has been carried out by Asthana and Agrawal [16]. They observed that the heat and mass transfer has a stabilizing effect when the lower fluid viscosity is high and destabilizing effect when fluid viscosity is low. Kim et al. [17] investigated the capillary instability problem of vapor liquid system in an annular configuration with heat and mass transfer using viscous potential flow for axisymmetric disturbances. Awasthi and Agrawal [18] have studied the nonlinear effects on the capillary instability in the presence of heat and mass transfer and observed that heat and mass transfer phenomenon has stabilizing effect on the stability of the system in the nonlinear analysis.

The magnetohydrodynamic interfacial instability with heat and mass transfer is of fundamental importance in number of applications such as design of many types of contacting equipment, e.g., boilers, condensers, reactors, and others industrial and environmental processes. The capillary instability with heat and mass transfer in a magnetic field occurs in many practical applications such as electronic magnetic ink jet printer and fluid jet amplifier. Elhefnawy and Radwan [19] studied the stability of magnetic inviscid fluids in cylindrical geometry with heat and mass transfer across the interface. Elhefnawy [20] analyzed the stability of the magnetic fluids of cylindrical interface with heat and mass transfer and periodic radial field. Bubble formation in superposed magnetic fluids in the presence of heat and mass transfer has been studied by Gill et al. [21]. Lee [22] considered the nonlinear stability of magnetic fluids with heat and mass transfer and showed that nonlinearity increases the region of stability with heat and mass transfer.

Sirwah [23] applied viscous potential flow theory to study the Kelvin–Helmholtz instability in the presence of tangential magnetic field and observed that the tangential magnetic field plays a stabilizing role in the stability analysis. Sirwah [24] investigated the nonlinear stability analysis of magnetic fluids in the presence of constant normal magnetic field and observed that normal magnetic field has destabilizing effect on the stability of the system. Awasthi and Agrawal [25] have studied the effect of irrotational shearing stresses on the magnetohydrodynamic Kelvin–Helmholtz instability and observed that irrotational shearing stress and tangential magnetic field both have stabilizing effect.

Awasthi [26] studies the effect of axial electric field on the capillary instability in the presence of heat and mass transfer and observed that axial electric field decays the amplitudes of disturbance waves. The effect of electric field on the capillary instability in the porous media is considered by Awasthi [27] and found that medium porosity destabilize the interface.

The objective of the present work was to study the effect of axial magnetic field on the capillary instability of cylindrical interface when there is heat and mass transfer across the interface. Both the fluids are taken as incompressible, viscous and magnetic with different kinematic viscosities and permeabilities, which have not been considered earlier. A dispersion relation is derived and stability is discussed theoretically and numerically. The critical values of magnetic field and wave number are obtained. The system is unstable when the magnetic field is less than the critical value of magnetic field, otherwise it is stable. The effect of ratio of magnetic permeability $\varepsilon = \frac{\varepsilon_1}{\varepsilon_2}$ on growth rates is observed. Various neutral curves have been drawn to show the effect of various physical parameters such as magnetic field, Ohnesorge number, heat transfer capillary number, and kinematic viscosities, on the stability of the system.

2. Methodology

2.1. Problem Formulation

A system of two incompressible, viscous and magnetic fluids, separated by a cylindrical interface, is considered in an annular configuration as shown in Fig. 1. A cylindrical system of coordinates (r, θ, z) is assumed so that in the equilibrium state z-axis is the axis of symmetry of the system. The undisturbed cylindrical interface is taken at radius R. In the formulation the subscripts 1 and 2 denote variables associated with the fluid