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Material transport of a magnetizable fluid by surface perturbation

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ABSTRACT

Within the research for apedal, contour variable locomotion systems, the influence of an alternating magnetic field on the shape of the free surface of a magnetizable fluid (magnetic fluid) is studied. In the framework of the Stokes approximation, for the case where the amplitude of the alternating component of the applied magnetic field is much less than the magnitude of the permanent component, it is shown analytically that a periodical traveling applied magnetic field can generate a transport of the fluid in a prescribed direction. Numerical computations are performed to calculate and analyze the flow rate of the fluid as a function of the parameters of the field and the fluid. This effect can be used in fluid transporting engineering mini- and microsystems.

1. Introduction

Deformation of the free surface of a magnetic fluid can be used for creating flows with nonzero flow rates both in the fluid itself and in the ambient medium. Various peristaltic pumps based on magnetic fluids have been already used in medicine for pumping biological fluids, in particular, blood, since such pumps preserve the structure of the fluids to be pumped. A number of new designs for the peristaltic pumps based on magnetic fluids are proposed by [1–3]. In addition, the deformation of the free surface of a magnetic fluid subjected to a time variable inhomogeneous magnetic field can be used for creating propulsion devices, stepper motors [4–6] and various kinds of valves, flow-rate meters and breakers.

The peristaltic flow of a viscous, incompressible fluid layer on a horizontal substrate induced by a sinusoidal deformation wave traveling along the fluid surface was studied by [7]. The perturbation of the surface was defined kinematically and, therefore, the nature of forces causing perturbations like these was not discussed. For the case of a magnetic fluid, these perturbations can be realized by magnetic forces in an inhomogeneous magnetic field. These forces can create a traveling surface deformation wave in the fluid as was described previously.

In a homogeneous vertical magnetic field, the horizontal surface of a magnetostatic fluid layer on a horizontal substrate can become unstable at a certain critical value of the field [8]. Steady-state spikes can appear on the surface without the simultaneous occurrence of a flow inside the fluid. An inhomogeneous magnetostatic field also cannot create a flow of a magnetic fluid with a flow rate. However, a traveling inhomogeneous magnetic field induces a traveling surface wave, which affects a flow with a nonzero flow rate inside the magnetic fluid. This phenomenon was observed in experiments [9–11], which created traveling waves on the surface of a magnetic fluid with a temporally and spatially varying magnetic field. Either a flow of a nonzero flow rate [9] was observed in these experiments or a sloping surface of the magnetic fluid appeared [10,11].

Analytical studies of the flow in a magnetic fluid layer subjected to a traveling magnetic field were performed by [12,9] using the perfect fluid model. Viscosity was taken into account by [12], who investigated analytically the flow of an infinitely deep layer of a magnetic fluid in such a field.

Thin layered flows of heavy viscous magnetic fluids on horizontal or cylindrical substrates in traveling magnetic fields were investigated by [13,14]. The surface tension was taken into account. In these studies, closed-form expressions were obtained for the magnetic field that created a prescribed sinusoidal traveling wave on the surface of a thin layered magnetic fluid. The thickness of the layer was assumed small in comparison with the wavelength. The problem of determining the magnetic field for a prescribed flow can be regarded as the inverse problem. The direct problem, in which the flow of a thin layered heavy viscous

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magnetizable fluid on a horizontal substrate is determined for a given traveling periodic magnetic field, was solved analytically by [15]. In this study, analytical expressions for the average flow rate in the finite-thickness layer were obtained.

In the present paper, the problem of determining the 2D flow of a finite-thickness layer of a heavy viscous magnetic fluid in a traveling periodic magnetic field is solved analytically for the case where the amplitude of the magnetic field oscillations is small. The surface tension is taken into account. The problem is solved in the Stokes approximation under the assumption that the magnetic permeability of the fluid is constant. Analytical expressions are obtained for the velocity, pressure, and average flow rate of the fluid.

2. Statement of the problem

Consider a planar flow (2D flow) of an incompressible magnetizable fluid bounded from below by a horizontal impermeable plane, as depicted in Fig. 1. A periodic traveling magnetic field $\boldsymbol{H}_a^*$ is applied. Here $\boldsymbol{H}_a^*$ is a applied magnetic field when a magnetizable fluid is absent. The square of the applied magnetic field strength is defined by

$$H_a^{*2} = H_0^{*2}(z^*) + A^2(z^*)\sin(\zeta^*), \tag{1}$$

where

$$\zeta^* = k^*x^* - \omega^*t^*. \tag{2}$$

Here, H_0^{*2} is the static component of H_a^{*2}, A^2 is the amplitude of the superimposed periodic perturbation, k^* is the wavenumber, ω^* is the angular frequency, t^* is time, x^* and z^* are the horizontal and vertical coordinates, respectively. The asterisks denote the dimensional variables and parameters. When the magnetizable fluid is subjected to the magnetic field, its free surface is deformed and acquires the shape described by the equation $z^* = h^*(x^*, t^*)$. Let ν, ϱ, and μ denote the kinematic viscosity, the density, and the magnetic permeability of the fluid, respectively, $\boldsymbol{g}$ the acceleration due to gravity, and p_a the atmospheric pressure. All these parameters are assumed to be constant and $(\mu - 1) \ll 1$ (noninduction approximation). In what follows, all physical quantities are measured in CGS units.

2.1. Equations of motion and boundary conditions

In the dimensional variables, the equations of motion of a magnetizable incompressible fluid in a specified coordinate system can be represented as follows:

$$\rho\frac{dv_i^*}{dt^*} = \nabla_j p_{ij}^* + \rho F_i^*, \tag{3}$$

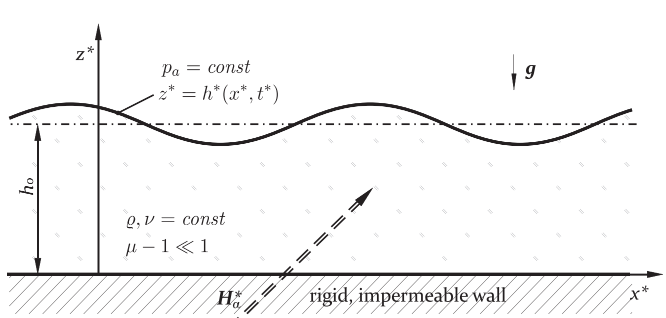

Fig. 1. Schematic of the system under consideration.

$$\nabla_i v_i^* = 0, \quad i, j = x, z, \tag{4}$$

where $v_x^* = u^*$ and $v_z^* = w^*$ are the horizontal and vertical components of the velocity vector $\boldsymbol{v}$, respectively, t^* is time, p_{ij}^* are the components of the stress tensor, $F_x^* = 0$, $F_z^* = -g$ is the mass density of the gravity force acting on the fluid, ∇_j stands for differentiation with respect to the corresponding coordinate, the summation with respect to doubly repeated indices is assumed. The stress tensor p_{ij}^* can be represented by

$$p_{ij}^* = -p^*\delta_{ij} + \tau_{ij}^* + p_{ij}^H, \tag{5}$$

where p^* is the fluid pressure, τ_{ij}^* is the viscous stress tensor, δ_{ij} is the Kronecker delta. The magnetic stress tensor p_{ij}^H has the form [8]

$$p_{ij}^H = \frac{H_i^*B_j^*}{4\pi} - \frac{H^*B^*}{8\pi}\delta_{ij}, \tag{6}$$

where H_i are the components of the magnetic field $\boldsymbol{H}^*$ when the magnetizable fluid distorts the applied magnetic field $\boldsymbol{H}_a^*$; B_j^* are the components of the vector $\boldsymbol{B}^*$ defined by $\boldsymbol{B}^* = \mu\boldsymbol{H}^*$ inside the magnetizable fluid and $\boldsymbol{B}^* = \boldsymbol{H}^*$ outside the magnetizable fluid.

Using Maxwell's equations div $\boldsymbol{B}^* = 0$ and rot $\boldsymbol{H}^* = 0$ we obtain

$$\nabla_j p_{ij}^H = -\frac{H^{*2}}{8\pi}\nabla_i\mu. \tag{7}$$

For fluids with constant μ this term disappears from (3), however, a surface force density $\boldsymbol{f_m}$ appears due to the step change in μ at the interface. The expression for $\boldsymbol{f_m}$ is [8]

$$\boldsymbol{f_m} = \left[-\frac{B_n^{*2}}{8\pi}\left(\frac{1}{\mu} - 1\right) + \frac{H_\tau^{*2}}{8\pi}\left(\mu - 1\right)\right]\mathbf{n} \quad \text{for } z^* = h^*(x^*, t^*). \tag{8}$$

Here B_n^* is the component of the vector $\boldsymbol{B}^*$ normal to the fluid surface, H_τ^* is the component of the magnetic field $\boldsymbol{H}^*$ tangential to the fluid surface (B_n^* and H_τ^* are continuous on the surface), $\mathbf{n}$ is the unit vector of the outer normal to the fluid surface. In the noninduction approximation, when $(\mu - 1) \ll 1$, we have $B_n^* = H_{an}^*(1 + O(\mu - 1)^2)$ and $H_\tau^* = H_{a\tau}^*(1 + O(\mu - 1)^2)$. Therefore, expression (8) for $\boldsymbol{f_m}$ can be written as

$$\boldsymbol{f_m} = \frac{H_a^{*2}}{8\pi}(\mu - 1 + O(\mu - 1)^2)\mathbf{n} \quad \text{for } z^* = h^*(x^*, t^*). \tag{9}$$

The dynamic condition at the fluid surface in the dimensional variables has the form

$$-p^*\mathbf{n} + \tau_{ij}^* n_j \mathbf{e}_i = \frac{\gamma\mathbf{n}}{R} + \boldsymbol{f_m} \quad \text{for } z^* = h^*(x^*, t^*), \tag{10}$$

$$R = \frac{(1 + h^{*2}_{'x^*})^{3/2}}{h^*_{'x^*x^*}}, \tag{11}$$

where R is the radius of curvature of the surface at the respective point, γ is the coefficient of surface tension, and $\mathbf{e}_i$ is the unit vector of the ith coordinate axis ($\mathbf{e}_1 = \mathbf{e}_x$, $\mathbf{e}_2 = \mathbf{e}_z$), the lower prime stands for differentiation with respect to the corresponding variable. The dynamic condition must be augmented by the kinematic condition at the fluid surface and the no-slip condition at the rigid base

$$\frac{dh^*}{dt^*} = w^*, \quad z^* = h^*(x^*, t^*), \tag{12}$$

$$u^* = w^* = 0, \quad z^* = 0. \tag{13}$$