Ain Shams Engineering Journal (2015) 6, 1203–1210

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

ENGINEERING PHYSICS AND MATHEMATICS

# Performance analysis of Arithmetic Mean method in determining peak junction temperature of semiconductor device

**Mohana Sundaram Muthuvalu** [a,b,*], **Vijanth Sagayan Asirvadam** [c], **Guwanch Mashadov** [c]

[a] *Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, 32610 Bandar Seri Iskandar, Perak, Malaysia*
[b] *Department of Petroleum Engineering, Universiti Teknologi PETRONAS, 32610 Bandar Seri Iskandar, Perak, Malaysia*
[c] *Department of Electrical and Electronics Engineering, Universiti Teknologi PETRONAS, 32610 Bandar Seri Iskandar, Perak, Malaysia*

Received 7 November 2014; accepted 17 April 2015
Available online 12 June 2015

**KEYWORDS**

Peak junction temperature;
Finite-difference method;
Arithmetic Mean method;
Performance analysis

**Abstract** High reliability users of microelectronic devices have been derating junction temperature and other critical stress parameters to improve device reliability and extend operating life. The reliability of a semiconductor is determined by junction temperature. This paper gives a useful analysis on mathematical approach which can be implemented to predict temperature of a silicon die. The problem could be modeled as heat conduction equation. In this study, numerical approach based on implicit scheme and Arithmetic Mean (AM) iterative method will be applied to solve the governing heat conduction equation. Numerical results are also included in order to assert the effectiveness of the proposed technique.

---

[*] Corresponding author at: Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, 32610 Bandar Seri Iskandar, Perak, Malaysia. Tel.: +60 10 9303005.
E-mail address: msmuthuvalu@gmail.com (M.S. Muthuvalu).

Peer review under responsibility of Ain Shams University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2015.04.007
2090-4479 © 2015 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V.

## 1. Introduction

High power is usually encountered in a power device application and it is important to make power devices reliable for their intended application. In order to achieve this goal, considerations have to be taken regarding reliability and performance. During the design phase, especially when a new platform for new technology is involved, thorough calculations and simulations are carried out to ensure the designed electrical parameters and other reliability characteristics are optimized. High reliability users of microelectronic devices have been derating junction temperature and other critical stress parameters for decades to improve device reliability and extend operating life [1]. It is in the first phase, i.e., design phase where semiconductor devices are stressed for reliability and performance [2] and it is very important to predict junction temperature at this phase. Consequently, corresponding electrical circuits as thermal modeling are widely applied because of their easy application in circuit simulators.

The present paper gives a performance analysis of the finite-difference method (FDM) with Arithmetic Mean (AM) iterative method in determining peak junction temperature of semiconductor device. Previously, the AM method has been applied extensively for solving various types of matrix equations problems. The effectiveness of the AM method and its variants were studied and tested on linear and nonlinear systems, refer [3–6] for recent papers.

The rest of this paper is organized in the following way. The mathematical modeling and numerical approach to determine peak junction temperature of semiconductor device will be elaborated in Sections 2 and 3 respectively. In Section 4, some simulation results are included. The discussions and concluding remark are given in Section 5.

## 2. Mathematical modeling

The following one-dimensional heat conduction equation is considered in modeling the thermal control system

$$K\frac{\partial^2 T(x,t)}{\partial x^2} = \rho c \frac{\partial T(x,t)}{\partial t} \tag{1}$$

since the thermal characteristics of silicon are assumed to be independent of temperature [7]. The $T, K, \rho$ and $c$ represent the absolute temperature, thermal conductivity of the semiconductor device (silicon), mass density of silicon and specific heat of silicon respectively. An Eq. (1) satisfies the following boundary conditions

$$\left.\begin{array}{l} SK\frac{\partial T}{\partial x}\Big|_{x=0} = -P_{in} \\ T(L,t) = T_{in} \end{array}\right\} \tag{2}$$

where $S, P_{in}, L$ and $T_{in}$ are surface of silicon, input power, thickness of vertical power device and input temperature respectively.

Heat is generated at the top surface of silicon and flows linearly along the $x$-axis which is perpendicular to the silicon surface, $S$. Thus, the top surface is considered to be a geometrical boundary of the device at $x = 0$ and the input power is assumed to be uniformly dissipated. Meanwhile, the lower surface i.e. at $x = L$ is considered to be the cooling boundary and the temperature is assumed to be equal to the input temperature, $T_{in}$. Also, the convection and radiation are assumed to be negligible.

## 3. Numerical approach

In this paper, numerical approach based on implicit scheme and AM iterative method will be considered. The following subsections will explain in detail the application of the numerical approach.

### 3.1. BTCS discretization scheme

As aforementioned, in this paper, FDM based on implicit scheme i.e. Backward Time, Centered Space (BTCS) is utilized in order to construct algebraic equations for problem (1). Now, let the solution domain be partitioned uniformly in both $x$ and $t$. Thus, the discrete set of points of $x$ and $t$, respectively, be given by $x_i = i\Delta x \quad (i = 0, 1, 2, \ldots, n-2, n-1, n)$ and $t_j = j\Delta t \ (j = 0, 1, 2, \ldots, m-2, m-1, m)$ where

$$\Delta x = \frac{L}{n} \tag{3}$$

and

$$\Delta t = \frac{t}{m}. \tag{4}$$

For simplicity, the following notation i.e., $T_{i,j} \equiv T(x_i, t_j)$ will be applied subsequently.

By using BTCS scheme

$$\frac{\partial T}{\partial t} = \frac{T_{i,j+1} - T_{i,j}}{\Delta t} + O(\Delta t) \tag{5}$$

and

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i-1,j+1} - 2T_{i,j+1} + T_{i+1,j+1}}{(\Delta x)^2} + O(\Delta x^2). \tag{6}$$

By substituting formulae (5) and (6) (by dropping the truncation error terms), an application of the BTCS scheme reduces problem (1) to

$$K\frac{T_{i-1,j+1} - 2T_{i,j+1} + T_{i+1,j+1}}{(\Delta x)^2} = \rho c \frac{T_{i,j+1} - T_{i,j}}{\Delta t} \tag{7}$$

which can be rewritten as follows

$$-\alpha T_{i-1,j+1} + \beta T_{i,j+1} - \alpha T_{i+1,j+1} = \gamma T_{i,j} \tag{8}$$

with $\alpha = \frac{K}{(\Delta x)^2}, \beta = \frac{2K}{(\Delta x)^2} + \frac{\rho c}{\Delta t}$ and $\gamma = \frac{\rho c}{\Delta t}$.

An implementation of the BTCS scheme requires solving a linear system at each time step.

Whereas first order discretization of the boundary condition gives

$$SK\frac{T_{1,j+1} - T_{0,j+1}}{\Delta x} = -P_{in} \tag{9}$$