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# Electro-magneto-thermo-elastic response of infinite functionally graded cylinders without energy dissipation

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ABSTRACT

The electro-magneto-thermo-elastic analysis problem of an infinite functionally graded (FG) hollow cylinder is studied in the context of Green–Naghdi's (G–N) generalized thermoelasticity theory (without energy dissipation). Material properties are assumed to be graded in the radial direction according to a novel power-law distribution in terms of the volume fractions of the metal and ceramic constituents. The inner surface of the FG cylinder is pure metal whereas the outer surface is pure ceramic. The equations of motion and the heat-conduction equation are used to derive the governing second-order differential equations. A finite element scheme is presented for the numerical purpose. The system of differential equations is solved numerically and some plots for displacement, radial and electromagnetic stresses, and temperature are presented. The radial displacement, mechanical stresses and temperature as well as the electromagnetic stress are all investigated along the radial direction of the infinite cylinder.


## 1. Introduction

The classical theory of thermoelasticity has an assumption of infinite speed, which is contrary to physical observation. Various generalized theories of thermoelasticity were developed to replace the classical one of heat conduction in solids. The most important generalized theories of thermoelasticity are Lord and Shulman's (L–S) theory [1], Green and Lindsay's (G–L) theory [2], and Green and Naghdi's (G–N) theory [3–5]. The first two theories (L–S and G–L) introduced one or two relaxation time in the thermoelastic process to eliminate the paradox of infinite speed for the propagation of thermal signals. They also involved a hyperbolic-type heat equation. They are structurally different and one cannot be obtained as a particular case of the other. The third generalized thermoelasticity theory of Green and Naghdi presented the governing thermoelasticity equations in three models. They obtained coupled equations in displacement and temperature fields based on finite wave speed.

The theory of electro-magneto-thermoelasticity is concerned with the interacting effects of the applied electromagnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in many industrial applications, particularly in nuclear devices, where there exists a primary magnetic field. Various investigations are to be carried out by considering the interactions among electric, magnetic, thermal and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic and electric studies. The development of the interactions of an electromagnetic field, the thermal field, and the elastic field is available in many studies. Recently, Zenkour and his colleagues [6–10] have presented the analysis of functionally graded piezoelectric cylinders and plates in a hygrothermal environment.

The theory of thermoelasticity without energy dissipation of Green and Naghdi [4] includes the “thermal displacement gradient” among its independent constitutive variables, and differs from the previous theories in that it does not accommodate dissipation of thermal energy. The propagation of thermoelastic waves in different structures is studied on the basis of Green and Naghdi's (G–N) generalized thermoelastic theory (without energy dissipation). Several investigations relating to thermoelasticity without energy dissipation theory have been presented by Chandrasekharaiah [11,12], Sharma and Chouhan [13], Roychoudhuri and Bandyopadhyay [14], Roychoudhuri and Dutta [15]. The coupled thermo-elasticity based on the G–N theory without energy dissipation is developed for infinite and finite functionally graded (FG) thick hollow cylinder using hybrid Galerkin finite element

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and Newmark finite difference methods by Hosseini and co-workers [16,17]. The propagation of thermoelastic waves in orthotropic spherical curved plates subjected to stress-free, isothermal boundary conditions is investigated in the context of the G–N generalized thermoelastic theory without energy dissipation [18]. Yu et al. [19] have investigated the wave propagation in circumferential direction of transversely isotropic cylindrical curved plates.

In the present paper, we have considered the electro-magneto-thermo-elastic problem which involved a hyperbolic type heat equation. The generation of radial displacement, mechanical stresses and temperature as well as the electromagnetic stress in an infinite FG elastic cylinder placed in a constant primary magnetic field are discussed. The governing equations are transformed into dimensionless forms and their solutions are obtained using the finite element approach. The outline of the coupled finite element solutions procedure gives the values of displacement and temperature using initial conditions. Numerical results are presented for the variation of the temperature, displacement, and stresses with the time and along the radial direction of the FG hollow cylinder. The effect of FG parameter is also investigated.

## 2. Formulation of the problem

Let us consider a long cylinder made of functionally graded material. The cylindrical coordinates system $(r, \theta, z)$ is used with $z$-axis coinciding with the axis of the cylinder. The material properties of the FG cylinder are assumed to be function of the volume fraction of the constituent materials. The functionally graded between the physical properties and the radial direction $r$ for ceramic and metal FG cylinder is given by [20–22]

$$P(r) = P_c + (P_m - P_c)\left(\frac{r-b}{a-b}\right)^n, \tag{1}$$

where $P_c$ and $P_m$ are the corresponding properties of ceramic (outer surface) and metal (inner surface), respectively, $a$ is the inner radius and $b$ is the outer radius of the cylinder. Note that, the parameter $n$ is the volume fraction exponent which takes positive real values. The value of $n$ equal to zero represents a fully metal cylinder. According to this distribution, the inner surface $(r = a)$ of the FG cylinder is pure metal whereas the outer surface $(r = b)$ is pure ceramic.

The strain axis is considered to be symmetric about the $z$-axis. We have only the radial displacement $u_r \equiv u$ which is independent of $\theta$ and $z$. In a generalized plane strain, we suppose that the planes perpendicular to the $z$-axis and $u_r$ is a function of the radial direction $r$ and the time $t$ only. So, the components of the stain tensor $e_{ij}$ are given by

$$e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\theta\theta} = \frac{u}{r}, \quad e_{zz} = 0. \tag{2}$$

The cylinder is placed in a constant primary magnetic field $H_0$. The medium is assumed to be non-ferromagnetic and ferroelectric. Neglecting the Thompson effect, the simplified Maxwell's equations of electro-dynamics for perfectly conducting elastic medium are as follows:

$$\nabla \times \vec{h} = \vec{j}, \quad \nabla \times \vec{E} = -\eta\frac{\partial \vec{h}}{\partial t}, \quad \nabla\cdot\vec{h} = 0, \quad \nabla\cdot\vec{E} = 0, \tag{3}$$

where

$$\vec{E} = -\eta\left(\frac{\partial \vec{u}}{\partial t} \times \vec{H}\right), \quad \vec{h} = \nabla \times (\vec{u} \times \vec{H}), \tag{4}$$

in which $\vec{H}$ is the magnetic field, $\vec{E}$ is the electric field, $\vec{j}$ is the current density, $\vec{u}$ is the mechanical displacement field, $\vec{h}$ is the perturbed magnetic, and $\eta$ is the magnetic permeability.

The equations of motion in the absence of the body forces are

$$\sigma_{ij,j} + \tau_{ij,j} = \rho(r)\ddot{u}_i, \tag{5}$$

where $\rho$ is the material density of the cylinder and it is also considered to be a function of $r$. The symbol $(\ )_{,j}$ means differentiation with respect to $x_j$. The mechanical stress tensor $\sigma_{ij}$ and Maxwell's electromagnetic stress tensor $\tau_{ij}$ are given, respectively, by

$$\sigma_{ij} = (\lambda e_{ii} - \gamma\Delta T)\delta_{ij} + 2\mu e_{ij},$$
$$\tau_{ij} = \eta\left(h_i H_j + h_j H_i - h_k H_k \delta_{ij}\right), \tag{6}$$

where $\delta_{ij}$ is Kronecker's delta function, $\Delta T = T - T_0$ in which $T_0$ is the absolute temperature, $\lambda$ and $\mu$ are Lamé's coefficients, $\eta$ is the magnetic permeability, $\gamma = (3\lambda + 2\mu)\alpha$ is the stress temperature modulus, in which $\alpha$ is the linear thermal expansion.

The magneto-elasto-dynamic equation, Eq. (5), in the radial direction of the FG hollow cylinder is given by

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + f_r = \rho(r)\frac{\partial^2 u_r}{\partial t^2}, \tag{7}$$

where

$$f_r = \frac{\partial \tau_{rr}}{\partial r}, \tag{8}$$

is defined as Lorentz's force, and

$$\left.\begin{aligned}
\sigma_{rr} &= \lambda(r)\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + 2\mu(r)\frac{\partial u}{\partial r} - \gamma(r)(T - T_0),\\
\sigma_{\theta\theta} &= \lambda(r)\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + 2\mu(r)\frac{u}{r} - \gamma(r)(T - T_0),\\
\tau_{rr} &= \eta(r)H_0^2\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right).
\end{aligned}\right\} \tag{9}$$

The heat conduction equation according to Green and Naghdi's theory [3–5] is given by

$$\frac{\partial^2 T}{\partial r^2} + \left(\frac{1}{r} + \frac{1}{K^*(r)}\frac{dK^*}{dr}\right)\frac{\partial T}{\partial r} = \frac{1}{K^*(r)}\frac{\partial^2}{\partial t^2}\left[c^e(r)\rho(r)T + \gamma(r)T_0\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right)\right], \tag{10}$$

where $c^e$ is the specific heat at constant strain, and $K^*$ is the material constant characteristics of the theory (the thermal conductivity). Generally, this study assumes that $\lambda$, $\mu$, $\gamma$, $K^*$, $\eta$, $c^e$ and $\rho$ of the FG cylinder change continuously through the radial direction of the hollow cylinder and obey the gradation relation given in Eq. (1).

## 3. Solution of the problem

Introducing the following dimensionless variables may be simplifying the solving process

$$(R, u^\circ, A, t^\circ) = \frac{1}{b}(r, u, a, ct), \quad c = \sqrt{\frac{\lambda_m + 2\mu_m}{\rho_m}},$$
$$(\sigma^\circ_{rr}, \sigma^\circ_{\theta\theta}, \tau^\circ_{rr}) = \frac{1}{\lambda_m + 2\mu_m}(\sigma_{rr}, \sigma_{\theta\theta}, \tau_{rr}), \quad T^\circ = \frac{T - T_0}{T_0}. \tag{11}$$

Here and in what follows the sup-index '$m$' denotes quantities for the homogeneous metal material while the sup-index '$c$' denotes quantities for the homogeneous ceramic material. The