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MHD stagnation point flow of Carreau fluid toward a permeable shrinking sheet: Dual solutions

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Abstract Present analysis is carried out to study the two-dimensional stagnation-point flow of an in-compressible Carreau fluid toward a shrinking surface. The formulation of the Carreau fluid model has been developed first time for boundary layer problem of shrinking sheet and the governing partial differential equations are rehabilitated into ordinary differential equations using similarity transformations. The simplified nonlinear boundary value problem is solved by Runge-Kutta method after converting into the system of initial value problem using shooting method. Dual solutions are obtained graphically and results are shown for various parameters involved in the flow equations. Numerical values of skin friction coefficients are also computed.

1. Introduction

A stagnation point occurs whenever a flow impinges on a solid object. The pioneer work for a two dimensional stagnation point flow was done by Hiemenz [1]. He discussed the 2-dimensional flow of a fluid near a stagnation point. He exposed that the Navier–Stokes equations governing the flow can be reduced to an ordinary differential equation of third order using similarity transformation. The study of boundary layer flow over a stretching sheet is a topic of great attention due to a variety of applications in designing cooling system which includes liquid metals, MHD generators, accelerators, pumps and flow meters. At very start Sakiadis [2] examined the laminar boundary-layer behavior on a moving continuous flat surface and he used similarity transformations to simplified boundary-layer equations and then solved numerically. Crane [3] extended the work of Sakiadis [2] for linear and exponential stretching. The steady two-dimensional stagnation point flow of an incompressible micropolar fluid over a stretching sheet when the sheet is stretched in its own plane with a velocity proportional to the distance from the stagnation point has been

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studied by Nazar et al. [4]. They solved the resulting non-linear ordinary coupled equations numerically using the Keller-box method. Two-dimensional stagnation-point flow of viscoelastic fluids is studied theoretically by Sadeghy et al. [5]. They assume that the fluid obeys the upper-convected Maxwell (UCM) model. Boundary-layer theory is used to simplify the equations of motion which are further reduced to a single non-linear third-order ODE using the concept of stream function coupled with the technique of the similarity solution. The obtained governing equation was solved using Chebyshev pseudo-spectral collocation-point method. The steady MHD mixed convection flow of a viscoelastic fluid in the vicinity of two-dimensional stagnation point with magnetic field has been investigated by Kumari and Nath [6]. They used upper-convected Maxwell (UCM) fluid as the proposed model. Boundary layer theory is used to simplify the equations of motion, induced magnetic field and energy which results in three coupled non-linear ordinary differential. These equations have been solved by using finite difference method. Ishak et al. [7] analyzed heat transfer over a stretching surface with uniform or variable heat flux in micropolar fluid. In this contest Nadeem and Hussain [8] discussed HAM solutions for boundary layer flow in the region of the stagnation point toward a stretching sheet. Nadeem et al. [9,10] developed the two and three dimensional boundary layer flow over stretching sheet for both Newtonian and non-Newtonian fluid. Analysis for MHD flow of a Maxwell fluid past a vertical stretching sheet in the presence of thermophoresis and chemical reaction was examined by Noor [11]. Recently Akbar et al [12] present the investigation on Magnetohydrodynamic boundary layer flow of tangent hyperbolic fluid toward a stretching sheet. In another article Nadeem and Haq [13] coated the effect of thermal radiation for MHD boundary layer flow of a nanofluid over a stretching sheet with convective boundary conditions. Stability of dual solutions in stagnation-point flow and heat transfer over a porous shrinking sheet with thermal radiation is given by Mahapatra and Nandy [14]. Later on many problems have been discussed by few authors [15–20].

In the present article model of Carreau fluid flow on a stretching sheet has been constructed along with the magnetic effects. To the best of author's knowledge no investigation has been done before in which Carreau fluid is model for shrinking/stretching sheet problems. The main objective of the article is to discuss the dual solution for MHD flow of Carreau fluid analysis on a stretching sheet. The formulation of the paper is organized as follows. The problem formulation is given in section two. The numerical solutions graphically with physical interpretation are presented in section three. Section four contains the conclusions of the current development.

2. Mathematical formulation

We discussed a two dimensional stagnation point flow of an incompressible Carreau fluid over a wall coinciding with plane $y = 0$, the flow is being confined to $y > 0$. The flow is generated due to the linear stretching. Extra stress tensor for Carreau fluid is [15],

$$\bar{\tau}_{ij} = \eta_o\left[1 + \frac{(n-1)}{2}(\Gamma\bar{\dot{\gamma}})^2\right]\bar{\dot{\gamma}}_{ij} \tag{1}$$

in which $\bar{\tau}_{ij}$ is the extra stress tensor, η_o is the zero shear rate viscosity, Γ is the time constant, n is the power law index and $\bar{\dot{\gamma}}$ is defined as

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i\sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\Pi}. \tag{2}$$

Here Π is the second invariant strain tensor. Flow equations for Carreau fluid model after applying the boundary layer approximations can be defined as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \nu\frac{3(n-1)\Gamma^2}{2}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}(u_e - u) + u_e\frac{\partial u_e}{\partial x} \tag{4}$$

Here u and v are velocity components along x and y direction, respectively. Where ν is kinematic viscosity, σ is the electrical conductivity, ρ is the density. It is noticed that for power law index ($n = 1$) our problem reduced to the case of Newtonian fluid while for $n > 1$ phenomena remains for non-Newtonian fluid. The corresponding boundary conditions are

$$\begin{aligned} &u = u_w(x) = ax, \quad v = v_w(x), \quad at \quad y = 0, \\ &u \to u_e(x) = bx, \quad as \qquad\qquad\quad y \to \infty, \end{aligned} \tag{5}$$

in which $b > 0$ is constant, we assume that $u_w(x) = ax$ and $u_e(x) = bx$ are the velocities near and away from the wall respectively. Introducing the following similarity transformations

$$\eta = \sqrt{\frac{b}{\nu}}y, \ \psi = \sqrt{b\nu}xf(\eta), \tag{6}$$

where η is the similarity variable and ψ is the Stream function defined in the usual notation as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfy the equation of continuity define in Eq. (3). By using above similarity transformation defined in Eq. (5) on Eqs. (2)–(4), we get:

$$f''' - (f')^2 + ff'' + 1 + \frac{3(n-1)We^2}{2}f'''(f'')^2 + M^2(1 - f') = 0 \tag{7}$$

$$f = s, f' = \lambda, at \quad \eta = 0 \tag{8}$$

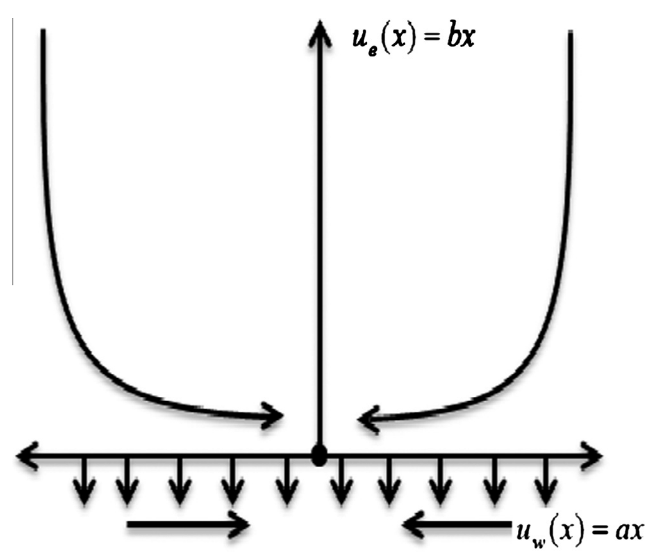

Figure 1 Geometry of the problem.