Ain Shams Engineering Journal (2014) 5, 1287–1297

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

ENGINEERING PHYSICS AND MATHEMATICS

Effect of rotational speed modulation on heat transport in a fluid layer with temperature dependent viscosity and internal heat source

B.S. Bhadauria [a,b,*], **Palle Kiran** [a]

[a] *Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow 226 025, India*
[b] *Departmentof Mathematics, Faculty of Sciences, Banaras Hindu University, Varanasi 221005, India*

Received 12 February 2014; revised 3 May 2014; accepted 21 May 2014
Available online 1 July 2014

KEYWORDS

Rotation speed modulation;
Coriolis force;
Internal Rayleigh number;
Taylor number;
Thermo-rheological parameter;
Nonlinear theory

Abstract In this paper, a theoretical investigation has been carried out to study the combined effect of rotation speed modulation and internal heating on thermal instability in a temperature dependent viscous horizontal fluid layer. Rayleigh–Bénard momentum equation with Coriolis term has been considered to describe the convective flow. The system is rotating about it is own axis with non-uniform rotational speed. In particular, a time-periodic and sinusoidally varying rotational speed has been considered. A weak nonlinear stability analysis is performed to find the effect of modulation on heat transport. Nusselt number is obtained in terms of amplitude of convection and internal Rayleigh number, and depicted graphically for showing the effects of various parameters of the system. The effect of modulated rotation speed is found to have a stabilizing effect for different values of modulation frequency. Further, internal heating and thermo-rheological parameters are found to destabilize the system.

1. Introduction

The classical Rayleigh–Bénard convection due to bottom heating is widely known and is a highly explored phenomenon, given in two excellent books of Chandrasekhar [1], Drazin and Reid [2]. Over decades, researchers are trying to explore various possibilities of controlling the convective phenomena in a horizontal fluid layer by imposing external physical constraints such as modulation which includes temperature, gravity, and magnetic field and rotation. Motivated by the experiments of Donnelly [3] about the effect of rotation speed modulation on the onset of instability in a fluid flow between two concentric cylinders, Venezian [4] performed a linear sta-

* Corresponding author at: Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow 226 025, India. Tel.: +91 522 2998127.
E-mail addresses: mathsbsb@yahoo.com (B.S. Bhadauria), kiran40p@gmail.com (P. Kiran).
Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

2090-4479 © 2014 Production and hosting by Elsevier B.V. on behalf of Ain Shams University.
http://dx.doi.org/10.1016/j.asej.2014.05.005

Nomenclature

Latin symbols

A	amplitude of convection
δ	amplitude of modulation
d	depth of the fluid layer
$\vec{q}$	velocity vector (U, V, W)
$\vec{g}$	acceleration due to gravity
k_c	critical wave number
Nu	Nusselt number
p	reduced pressure
V_T	thermo-rheological parameter $V_T = \delta_0 \Delta T$
R_i	internal Rayleigh number $R_i = \frac{Qd^2}{\kappa_T}$
Ra_T	thermal Rayleigh number $Ra_T = \frac{\alpha_T g \Delta T d^3}{\nu \kappa_T}$
R_{0c}	critical Rayleigh number
Ta	Taylor number $Ta = \left(\frac{2\Omega d^2}{\nu}\right)^2$
T	temperature
Pr	Prandtl number $Pr = \frac{\nu}{\kappa_T}$
ΔT	temperature difference across the fluid layer
t	time
(x, z)	horizontal and vertical co-ordinates

Greek symbols

α_T	coefficient of thermal expansion
β^2	square of horizontal wave number $\beta^2 = k_c^2 + \pi^2$
ϵ	perturbation parameter
κ_T	effective thermal diffusivity
ω	frequency of modulation
$\vec{\Omega}$	rotation speed vector $(0, 0, \Omega(t))$
μ	dynamic viscosity of the fluid
ν	kinematic viscosity $\left(\frac{\mu}{\rho_0}\right)$
ρ	fluid density
ψ	stream function
τ	slow time $\tau = \epsilon^2 t$

Other symbols

∇_1^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
∇^2	$\nabla_1^2 + \frac{\partial^2}{\partial z^2}$

Subscripts

b	basic state
c	critical
0	reference value

Superscripts

$'$	perturbed quantity
$*$	dimensionless quantity

bility analysis of Rayleigh–Bénard convection under temperature modulation with free–free surfaces. He obtained correction in critical Rayleigh number due to modulation, and suggested that by suitably tuning the frequency of modulation one can regulate the heat transport effectively. Later on, Rayleigh–Bénard problem under temperature modulation was studied by various researchers, considering different physical models [5–19].

A brief study of the combined effect of thermal modulation and rotation on the onset of convection in a rotating fluid layer was made by Rauscher and Kelly [20]. Liu and Ecke [21] analyzed heat transport of turbulent Rayleigh–Bénard convection under rotational effect. Malashetty and Swamy [22] investigated thermal instability of a heated fluid layer subjected to both boundary temperature modulation and rotation. They found that by proper tuning of modulation frequency, Taylor number and Prandtl number, it is possible to advance or delay the onset of convection. Kloosterziel and Carnevale [23] investigated the effect of rotation on the stability of thermally modulated system, and determined analytically the critical points on the marginal stability boundary above which an increment either in viscosity or in diffusivity is destabilizing. Finally, they showed that when the fluid has zero viscosity the system is always unstable, in contradiction to Chandrasekhar [1] conclusion. Malashetty and Swamy [24] studied the effect of rotation on the stability of thermally modulated system. They found that the symmetric modulation destabilizes the system at low frequencies while it stabilizes at moderate and high frequencies. Asymmetric modulation is the most stable situations for all frequencies. Lower wall temperature has stabilizing effect for lower and higher values of frequency and destabilizing effect for moderate values of frequency. Bhadauria [25] investigated rotational influence on Darcy convection and found that both rotation and permeability suppress the onset of thermal instability. Bhadauria et al. [26] investigated the nonlinear thermal instability in a rotating viscous fluid layer under temperature/gravity Modulation. They found that by suitably adjusting the frequency or amplitude of modulation one can control the convective flow.

Although rotation speed modulation was the originating idea of the temperature as well as of gravity modulation, but the research work in this field is scarce. The effect of temperature modulation on the Rayleigh–Bénard instability and the effect of modulation of the rotation speed in the Taylor–Couette instability have been investigated in detail both theoretically and experimentally by Ahlers et al. [9], Niemela and Donnelly [27], Kumar et al. [28], Meyer et al. [29], Walsh and Donnelly [30]. For Rayleigh–Bénard convection, the temperature modulation is supposed to stabilize the conduction state. However, since the temperature modulation breaks the reflection symmetry about the mid-plane and hexagons rather than cylinders, takes the pattern in which convection occurs immediately above the threshold. For the Rayleigh–Bénard problem with rotation, the above problem can be avoided if the rotation speed is modulated. This leads to a simple problem for the study of the effect of modulation on the threshold. When we study the rotation effect, one more parameter in the form of rotation speed appears, which can affect the stability of convective flow. Amongst the literature, the study by Bhattacharjee [31] is of great importance, in which he studied the effect of rotation speed modulation on Rayleigh–Bénard convection in an ordinary fluid layer. He found that the effect of modulation is stabilizing for most of the configurations. Suthar et al. [32] investigated the effect of the rotation speed