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Non-dipole interaction of helix inclusions in metamaterials with artificial permeability

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ABSTRACT

An analytic approach to calculate induced permeability of metamaterial filled with helix inclusions is developed. For an electrically small coil, the dynamic magnetization is determined by lumped inductance, capacitance, and resistance that are calculated from the coil design. The susceptibility of a coil is shown to be close to that of diamagnetic ellipsoid of the same elongation. Contrary to permeable particles in a composite, the coils screen each other thus decreasing the magnetic response of metamaterial compared to total response of comprised coils. The filling effect on magnetic spectrum of metamaterial is analyzed and verified by measurements for the case of identical coaxially placed coreless coils. The critical filling where magnetic response of metamaterial is maximal depends on coil shape and resistance. The coil-filled metamaterials may find applications as permeable EMI suppressors or microwave absorbers free of Snoek or Acher limitations on the high-frequency permeability, as well as of Curie limitation on the high-temperature magnetic performance.

1. Introduction

Metamaterials are composites containing artificial inclusions that may exhibit properties unattainable with natural fillers. The article is devoted to helix inclusions with dynamic magnetism induced by external electromagnetic field [1]. Metamaterials possessing induced magnetism may find applications as media with negative permeability [2], microwave absorbers, electromagnetic compatibility enhancers, etc. For these applications, the inclusions must exhibit maximal absorption cross-section (the effective area or radiation resistance if inclusion is considered as an electrically small antenna) as they are more difficult to prepare and more expensive than conventional powders or fibers. The advantage of helix inclusions over natural magnets is that parameters of magnetic absorption line may be readily tuned to the needs of particular applications; in addition, artificial magnetism is not limited by Curie temperature.

If the helix is multilayer, or its length and winding pitch are much smaller than helix diameter, the helix turns into a coil where electric field is locked inside the winding and the electric polarization and therefore chirality are negligible. A tightly wound coil has higher inductance and dynamic magnetization than a loosely wound helix of the same size. Besides, the coil winding is electrically small near the resonance frequency, because helix winding is about half of resonance wavelength. Therefore, the coil can be described using a simple lumped LCR-parameter approach.

The research aim is to describe the dependence of magnetic spectrum of metamaterial on winding parameters and volume fraction of regularly placed parallel coils. The analysis is limited to permeability of anisotropic composite with coreless coils, though the results are true for chiral helix-filled metamaterials and for coils with permeable cores as well.

Contrary to natural magnets, where circular currents arise due to electron motion, the current in helix is induced by external high-frequency magnetic field. Therefore, the nature of artificial magnetism in the helix inclusions is close to diamagnetism arising due to eddy currents. The electrically small coreless coil (a split ring resonator exhibits diamagnetism [3] and is a particular case of a single-turn coil) is diamagnetic like a thick metal particle; the high frequency magnetic field does not penetrate inside the coil, the same as inside a thick conductor. The difference is that the eddy current is replaced by ordered circular current in a resonant coil. Hence the self-capacitance loaded coil is an oscillating circuit with a Lorentz absorption line, while a thick conductive particle displays a broad absorption spectrum due to skinning [4]. The less obvious difference analyzed here is that contrary to composites

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with natural magnets, the interaction between adjacent coils is defined by flux linkage that decreases their magnetic response.

The methods to calculate the lumped LCR-parameters of a separate coil, to relate these to the susceptibility of equivalent elliptic inclusion and to the permeability of coil-filled composite are developed and verified experimentally.

2. Self-resonance parameters of a helix

Diamagnetic absorption of a composite filled with copper spheres has been studied in [4]. It is shown that magnetic absorption arises due to eddy currents and reaches a peak at the frequency where the skin depth and particle radius are equal. The shape of absorption spectrum resembles an asymmetrically distorted relaxation line. The diamagnetic behavior similar to that of a metal disk is observed in composites with shorted and split-rings [3]. Contrary to the gapless ring, the splitted loop displays narrow Lorentz-type absorption line. The split-ring is described in [3] as a resonance cavity, but if the loop size is much smaller than the wavelength, the simpler lumped-parameter LCR-circuit approach is true. The LCR approach is also valid for a multiturn coil, as the dimensions of a multiturn coil are smaller than that of a split-loop at the same resonance frequency. To calculate the resonance frequency of an unloaded solenoid (tightly wound long cylindrical helix), one needs to know its self-capacitance and inductance.

The inductance L of coil is calculated [5,6] as

$$L = \mu_0\left(\frac{\pi D^2 h}{4}\right)\left(\frac{n_{turn}}{h}\right)^2 \times \left(1+\frac{4}{3\pi}\frac{D}{h}\right)^{-1} \tag{1}$$

where $\mu_0 = 1.257\times10^{-6}$ Hn/m, D and h are coil diameter and length, and n_{turn} is the number of winding turns (SI units). The first two factors in (1) are a handbook formula for inductance of a long solenoid, The last factor is Wheeler's approximation [5] for the coil-shape factor, k_{shape}, sometimes referred to as the coil demagnetizing factor, N_{coil}. Eq. (1) is much simpler to calculate than the original Lorenz formula [6] and, at the same time, is accurate enough. Note that D in Eq. (1) is equal to geometric mean between external D_{max} and internal D_{min} coil diameters $D_{mean} = \sqrt{D_{max}D_{min}}$ only at low frequency, where wire diameter d_{wire} is smaller than the penetration depth. At high frequency and with thick wire, the accurate account for the proximity effect [7] is needed otherwise D is a fitted parameter: $D_{min} \le D \le D_{mean}$. The interloop flux leakage is also neglected in Eq. (1): wire diameter d_{wire} and winding pitch τ are considered negligible compared to the coil diameter, $d_{wire} \le \tau \ll D$, (the coil is tightly wound).

Fig. 1 plots the Landau–Lifshitz approximations for the coil-shape factor for short and long coils and Wheeler's dependence of coil inductance on coil shape [5]. The dashed black line shows the susceptibility for an ideal diamagnetic ($\mu=0$) ellipsoid of rotation as a function of ellipsoid elongation. A good agreement is seen between the coil-shape inductance factor and the susceptibility of diamagnetic ellipsoid. Therefore, the inductance and field structure of a cylindrical coil are close to that of a conductive ellipsoid of the same elongation h/D. Moreover, for both cases we calculate the shape of a permeable ellipsoid with the same susceptibility module $|\alpha|$ and field structure. A short coil and an oblate diamagnetic ($\mu=0$) ellipsoid have the same field structure as that of an ideal permeable ($\mu=\infty$) prolate ellipsoid [8]. The shape of a permeable ellipsoid possessing the absolute value of susceptibility $|\alpha|$ equal to that of a diamagnetic ellipsoid can be readily obtained numerically taking into account that both ellipsoids have the same demagnetizing factors: $N_{\mu=\infty} = 1 - N_{\mu=0} \approx 1 - N_{coil}$. Note that here we neglect the opposite susceptibility signs.

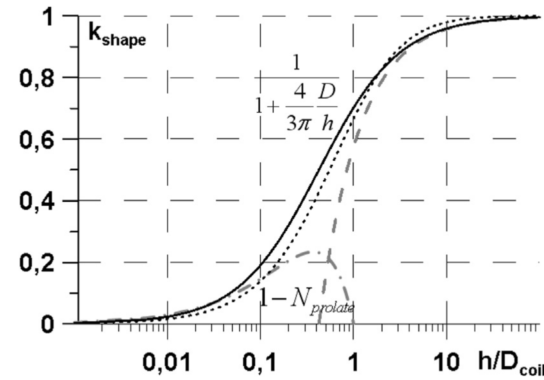

Fig. 1. The dependence of coil demagnetizing factor on the elongation (ratio of coil length to its diameter). The gray lines correspond to Landau–Lifshitz approximations for a long solenoid (the dashed line) and for a flat coil (the dash-dotted line). The solid black line shows Wheeler's approximation [5] for the Lorenz shape factor. The dotted black line shows the demagnetizing factor of a conductive ellipsoid [8].

The shapes of a short coil that approximates an oblate diamagnetic and prolate permeable ellipsoid with the same field pattern are shown in Fig. 2. Note that Eq. (1) gives the inductance of a cylindrical coil. If a loop has an ellipsoidal shape, the inductance derived from ellipsoid susceptibility [8] (as it is easy to show, the inductance is proportional to the magnetic susceptibility χ) slightly differs from Eq. (1):

$$L = \mu_0\left(\frac{\pi D^2 h}{6}\right)\left(\frac{1}{h}\right)^2 \times (1 - N_x) \tag{2}$$

The first factor in Eq. (2) is the inclusion volume and N_x is the demagnetizing factor for the ideal permeable ($\mu=\infty$) ellipsoid having the same elongation as that of the coil.

It is possible to compare the volume of diamagnetic ellipsoid given by Eq. (2) and that of a single-turn cylinder coil (split-ring or split-tube) given by Eq. (1) that have the same elongation and inductivity (equal shape and $|\alpha|$):

$$(1 + 4D/3\pi h)^{-1} = 2(1 - N_x)/3 \tag{3}$$

Solving this equation numerically we see that a prolate ellipsoid has the same volume as a long cylinder and an oblate ellipsoid has the volume about 1.5 times higher than that of a disk or a short cylinder, see Fig. 2.

Analyzing the frequency behavior of a single helix requires accounting for the self-capacitance, as the inductance is loaded by this very capacitance to form a resonance circuit.

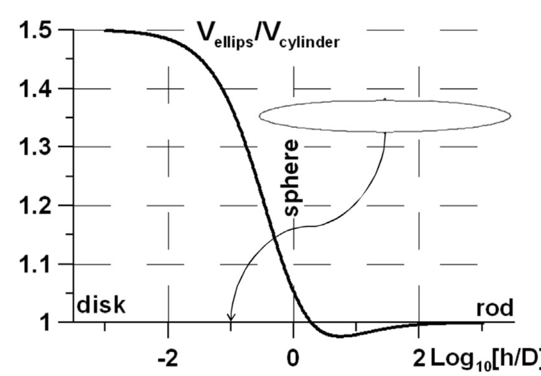

Fig. 2. The ratio of the volume of rotational ellipsoid to that of cylinder of the equal elongation plotted against the relative length. The insert shows a schematic approximation of a disk coil ($h/D=0.1$) by a prolate permeable ($\mu=\infty$) ellipsoid.

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