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Magnetic field effect on second order slip flow of nanofluid over a stretching/shrinking sheet with thermal radiation effect

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ABSTRACT

The magnetic field effect on a steady two dimensional laminar radiative flow of an incompressible viscous water based nanofluid over a stretching/shrinking sheet with second order slip boundary condition is investigated both analytically and numerically. The governing partial differential equations are reduced to nonlinear ordinary differential equations by means of Lie symmetry group transformations. The dimensionless governing equations for this investigation are solved analytically using hyper-geometric function and numerically by the fourth order Runge–Kutta method with the shooting technique. A unique exact solution exists for momentum equation in stretching sheet case and dual solutions are obtained for shrinking sheet case which has upper and lower branches. It is found that the lower branch solution vanishes in the presence of higher magnetic field. The velocity and temperature profiles, the local skin friction coefficient and the reduced Nusselt number are examined and discussed for different spherical nanoparticles such as Au, Ag, Cu, Al, Al_2O_3 and TiO_2. A comparative study between the previously published results and the present analytical and numerical results for a special case is found to be in good agreement.

1. Introduction

Nanofluids are suspensions of nanoparticles in fluids that show significant enhancement of their properties at modest nanoparticle concentrations. Many of the publications on nanofluids understand their behaviour so that they can be utilized where straight heat transfer enhancement is paramount as in many industrial applications, nuclear reactors, transportation, electronics as well as biomedicine and food. The broad range of current and future applications involving nanofluids have been given by Wong and Leon [1]. Nanofluids are not naturally occurring but they are synthesized in the laboratory. The thermal conductivity of these fluids plays an important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Therefore, numerous methods have been taken to improve the thermal conductivity of fluid by suspending nano/micro-sized particles' materials in base fluid such as oil, water and ethylene glycol mixture. Nanofluids enhance thermal conductivity of the base fluid enormously, which are also very stable and have no additional problems, such as sedimentation, erosion, additional pressure drop and non-Newtonian behaviour due to the tiny size of nanoelements and the low volume fraction of nanoelements required for conductivity enhancement. These suspended nanoparticles can change the transport and thermal properties of the base fluid [2–6].

Boundary layer flow over a stretching surface is an important problem in many engineering processes, for example, aerodynamic extrusion of plastic sheets, melt-spinning, the hot rolling, wire drawing, glass-fiber and paper production, manufacture of plastic and rubber sheets. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching. The nanoboundarylayer flows have been recently considered by several authors [7–15].

The study of magnetic field has important applications in medicine, physics and engineering. Many industrial types of equipment, such as MHD generators, pumps, bearings and boundary layer control, are affected by the interaction between the electrically conducting fluid and magnetic field. The behaviour of the flow strongly depends on orientation and intensity of the applied magnetic field. The exerted magnetic field manipulates the suspended particles and rearranges their concentration in the fluid which strongly changes heat transfer characteristics of the flow. The applied magnetic field plays an important role in controlling momentum and heat transfer in the boundary layer flow of different fluids over stretching/shrinking sheets. For this reason, many researchers have considered the magnetic field effects on

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the fluid flow field in different geometries [16–26]. Hamad [27] have done an analytical work on the natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field. Vishnu Ganesh et al. [28] analyzed the effect of magnetic field on nanofluid flow over a stretching sheet numerically. Govindaraju et al. [29] investigated the magnetic field effect on entropy generation of water based nanofluid flow over a stretching sheet analytically. Recently, the MHD flow of nanofluid over a stretching sheet in the presence of space and temperature dependent internal generation/absorption has been considered by Ganga et al. [30]. The effects of magnetic field along with the thermal radiation effects are also investigated by the followers in stretching sheet [31–35]. Vishnu Ganesh et al. [36] studied the effect of thermal radiation on MHD flow of water based metal nanofluids over a stretching sheet both analytically and numerically. Very recently, Rashidi et al. [37] studied the buoyancy effect on MHD flow of water based nanofluid over a stretching sheet with thermal radiation effect.

When the fluid is particulate such as emulsions, suspensions, foams and polymer solutions, the no slip condition is inadequate. The problem of flow and heat transfer of nanofluid over a stretching/shrinking surface with slip (Partial slip) regime has been investigated and discussed in [38–41]. Abdul Hakeem et al. [42] investigated the partial slip effect on MHD flow of a Newtonian fluid over a porous stretching sheet. Fang et al. [43] considered the effects of second order slip on ordinary fluid flow over a shrinking sheet and the case of stretching sheet was studied by Nandeppanavar et al [44]. Turkyilmazoglu [45] analyzed the second order slip flow and heat transfer over a stretching/shrinking sheet with magnetic field effect in ordinary fluid.

To the author's knowledge no studies have thus far been communicated with regard to second order slip MHD flow and convective heat transfer of a nanofluid over a stretching/shrinking sheet with thermal radiation effect. Keeping this in mind, in the present paper we have investigated the heat transfer of MHD radiative flow of nanofluid over a stretching/shrinking sheet with second order slip boundary condition.

2. Formulation of the problem

Consider the steady, two-dimensional laminar and radiative slip flow of an incompressible viscous water based nanofluid over a continuously stretching or shrinking sheet coinciding with the plane $\bar{y} = 0$ and the flow being confined to $\bar{y} > 0$ as shown in Fig. 1. We also consider the influence of a constant magnetic field of strength B_0 which is applied normally to the sheet. The temperature at the stretching/shrinking surface takes the constant value T_w, while the ambient value, attained as $\bar{y}$ tends to infinity, takes the constant value T_∞. It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field (as the magnetic Reynolds number is small). The fluid is water based nanofluid containing different types of spherical nanoparticles such as gold (Au), copper (Cu), silver(Ag), aluminium (Al), aluminium oxide (Al_2O_3) and titanium oxide (TiO_2). It is assumed that the nanofluid experiences a second order slip at the sheet surface and also it is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo-physical properties of the nanofluid are considered as given in Table 1. Taking the above assumptions into consideration, the steady boundary layer equations governing the flow and heat transfer of a nanofluid in the presence of transverse magnetic field and thermal radiation can be written as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \qquad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \left(\frac{\mu_{nf}}{\rho_{nf}}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2 \bar{u}}{\rho_{nf}}, \qquad (2)$$

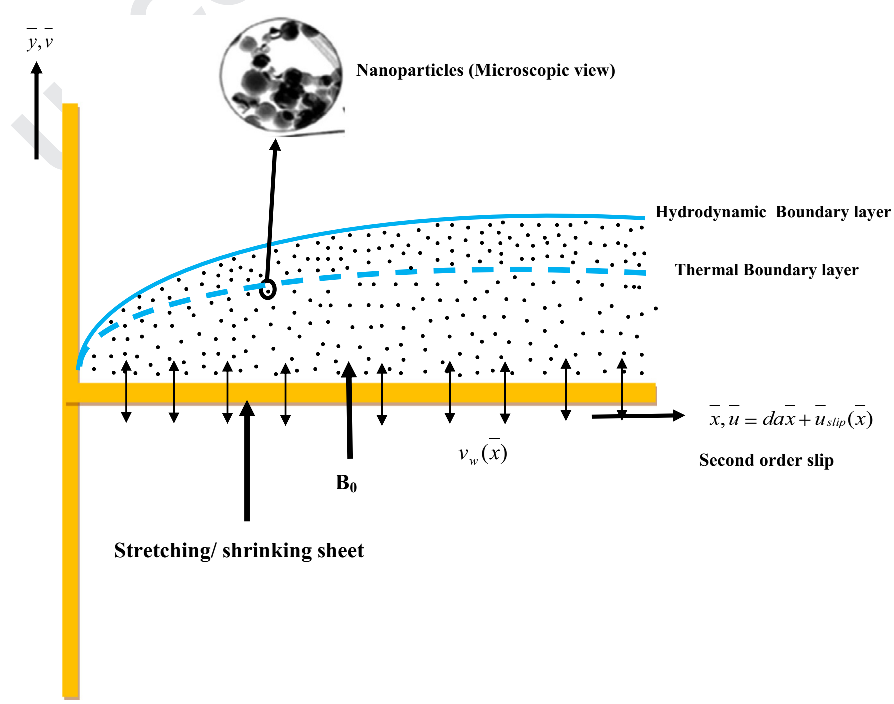

Fig. 1. Physical model and coordinate system of the problem.

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