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Magnetic field effects for copper suspended nanofluid venture through a composite stenosed arteries with permeable wall

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ABSTRACT

In the present paper magnetic field effects for copper nanoparticles for blood flow through composite stenosis in arteries with permeable wall are discussed. The copper nanoparticles for the blood flow with water as base fluid is not explored yet. The equations for the Cu–water nanofluid are developed first time in the literature and simplified using long wavelength and low Reynolds number assumptions. Exact solutions have been evaluated for velocity, pressure gradient, the solid volume fraction of the nanoparticles and temperature profile. The effect of various flow parameters on the flow and heat transfer characteristics is utilized.

1. Introduction

The study of blood flow in a stenosed artery is a very useful topic because of the fact that a number of cardiovascular diseases in the blood vessel such as hearts attacks and strokes are the leading cause of deaths. In cardiac related problems, the effected arteries get harden as a result of accumulation of fatty substances inside the lumen. These accumulations of substances in arteries are known as stenosis. The mathematical modelling of blood flow through an overlapping arterial stenosis is presented by Chakravarty and Mandal [1]. Then they give the theoretical study on the nonlinear behaviour of blood flow over a single cardiac cycle through an arterial segment having an overlapping stenosis with body acceleration and discussed the analysis through numerical computation see Chakravarty and Mandal [2]. A mathematical analysis for the flow of blood in a stenosed arterial segment is studied by Mishra and Chakravarty [3]. The mathematical modelling of pulsatile flow of Herschel Bulkely fluid in stenosed arteries has been examined by Sankar and Lee [4]. They used regular perturbation technique and found analytical solutions. The Newtonian behaviour of blood flow is also discussed in their article. The study on three layered oscillatory blood flow through stenosed arteries is discussed by Tripathi [5]. Mekheimer and Kot [6] have discussed the mathematical modelling of Sisko fluid through an anisotropically tapered elastic artery with time variant overlapping stenosis. Keeping blood flow as non-Newtonian fluids, Mishra et al. [7] have discussed the blood flow through a composite stenosis in an artery with permeable walls. The physiological importance of studies pertaining to the variation of the resistance to flow and the wall shear stress is discussed by Akbar [8,9]. Further recent survey related to the topic could be seen in Refs. [10,11].

Nanofluid has enormous input in industry and subsequently materials of nanometers measurement inspect unrivaled corporeal and chemical physiognomies. Water, ethylene glycol and oil are communal examples of base fluids used for the nanofluid singularity. Nanofluids have their huge applications in heat transfer, like microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines, domestic refrigerator, chiller, nuclear reactor coolant, and grinding and space technology, extensive literature is available which deals with the study of nanofluid and its applications [12–17]. Vajravelu et al. [18] focus on Ag–water and Cu–water nanofluids, and investigate the effects of the nanoparticle volume fraction on the flow and heat transfer characteristics under the influence of thermal buoyancy and temperature dependent internal heat generation or absorption. According to them the Ag–water solution decreases the boundary layer thickness more than that of the Cu–water solution. Recent work can be viewed by Refs. [19–30]

According to the authors knowledge the copper nanoparticles for the blood flow water as base fluid is not explored so far. To fill

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this gap we here discussed copper nanoparticles for blood flow through composite stenosis in arteries with permeable wall. In the next section we present formulation of the problem. Section 3 gives solutions of the problem. In Section 4 we discussed physical significance of the problem through graphs and discussion. Last section contains the summary of the present work.

2. Formulation of the problem

Consider an axisymmetric flow of blood through a composite stenosis in a circular tubule of finite length L, with permeable wall as shown in Fig. 1. The geometry of arterial wall with composite stenosis is described by Joshi and Srivastava [1,2] as

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{2\delta}{R_0\, L_0}(z-d); & d < z \le d + \frac{L_0}{2}, \\ 1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{2\pi}{L_0}\left(z - d - \frac{L_0}{2}\right)\right); & d + \frac{L_0}{2} < z \le d + L_0, \\ 1 & \text{otherwise.} \end{cases} \quad (1)$$

where $R(z)$ is the radius of the artery in the obstructed region while R_0 is the radius of normal artery. L_0, d, δ are the length, location and height of the stenosis respectively.

The governing equations for an incompressible nanofluid can be written as

$$\frac{1}{r}\frac{\partial(rv)}{\partial r} + \frac{\partial u}{\partial z} = 0,$$

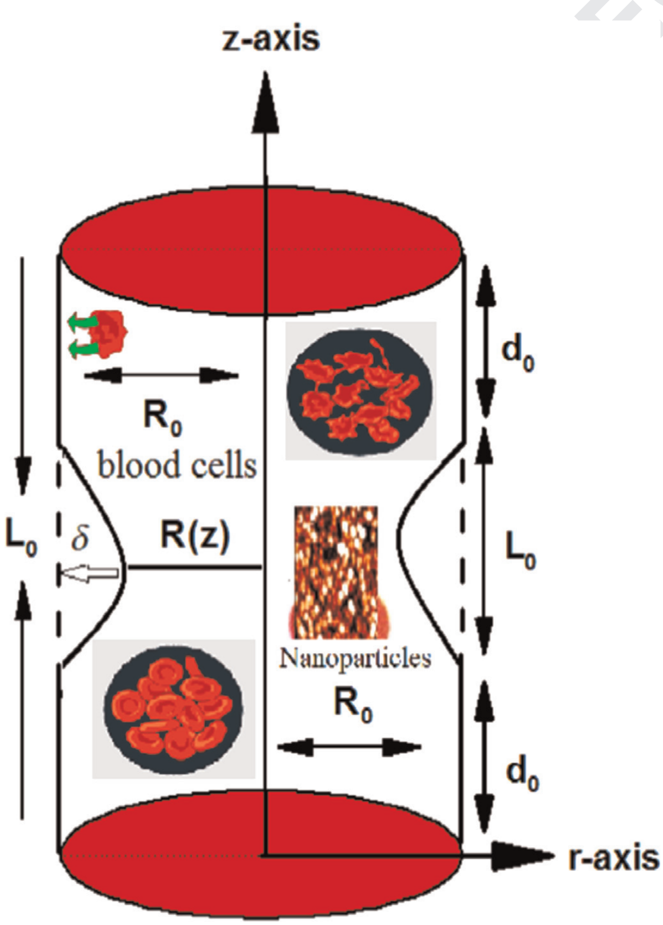

Fig. 1. Geometry of the problem.

$$\rho_{nf}\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu_{nf}\frac{\partial}{\partial r}\left(2\frac{\partial v}{\partial r}\right) + \mu_{nf}\frac{\partial}{\partial z}\left(2\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right), \quad (2)$$

$$\rho_{nf}\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu_{nf}\frac{\partial}{\partial z}\left(2\frac{\partial u}{\partial z}\right) + \frac{\mu_{nf}}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right)\right] \quad (3)$$

$$+\rho_{nf}\, g\alpha(\bar{T} - T_0) - \sigma B_0^2 u,$$

$$(\rho_{cp})_{nf}\left(v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z}\right) = k_{nf}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right] + Q_0. \quad (4)$$

with the conditions

$$\frac{\partial u}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0 \quad \text{at } r = 0, \quad (5a)$$

$$u = u_B, \quad T = \theta \quad \text{at } r = \frac{R(z)}{R_0}. \quad (5b)$$

where r and z are the coordinates. z is taken as the centre line of the tube and r transverse to it, u_B the slip velocity, u and v are the velocity components in the r and z directions respectively, T is the local temperature of the fluid. Further, ρ_{nf} is the effective density, μ_{nf} is the effective dynamic viscosity, $(\rho c_p)_{nf}$ is the heat capacitance, α_{nf} is the effective thermal diffusivity, and k_{nf} is the effective thermal conductivity of the nanofluid, which are defined as (see refs. [20–23])

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_f, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, (\rho c_p)_{nf}$$
$$= (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s,$$
$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, k_{nf} = k_f\left(\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}\right) \quad (6)$$

We introduce the following non-dimensional variables:

$$\bar{r} = \frac{r}{R_0}, \quad \bar{z} = \frac{z}{L_0}, \quad \bar{v} = \frac{L_0}{\delta U}v, \quad \bar{u} = \frac{u}{U},$$
$$\bar{d} = \frac{d}{L_0}, \quad \bar{R} = \frac{R}{R_0}, M^2 = \frac{\sigma B_0^2 R_0^2}{\mu_f}, \quad G_r = \frac{g\alpha R_0^2 T_0 \rho_{nf}}{U\mu_f}, \quad \bar{\delta} = \frac{\delta}{R_0}, \quad \theta = \frac{T - T_0}{T_0},$$
$$\bar{p} = \frac{UL_0\mu}{R_0^2}p, \quad \beta = \frac{Q_0 R_0^2}{k_f T_0}, \quad \mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}. \quad (7)$$

where U is the velocity averaged over the section of the tube with radius R_0. Making use of these variables in Eqs. (2)–(5)**?** and applying the additional condition $\epsilon = R_0/L_0 = o(1)$ for the case of mild stenosis ($\delta/R_0 \lll 1$), the non-dimensional governing equations after dropping the dashes can be written as

$$\frac{\partial p}{\partial r} = 0, \quad (8)$$

$$\frac{dp}{dz} = \frac{1}{(1-\varphi)^{2.5}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - M^2 u + G_r\theta, \quad (9)$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) + \beta\left(\frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)}\right) = 0, \quad (10)$$

where M, β and G_r are the Hartmann number, heat absorption parameter and Grashof number respectively. The non-dimensional boundary conditions on velocity and temperature for permeable wall are

$$\frac{\partial u}{\partial r} = 0, \quad \frac{\partial \theta}{\partial r} = 0 \quad \text{at } r = 0, \quad (11a)$$