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MHD squeezing flow between two infinite plates

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Abstract Magneto hydrodynamic (MHD) squeezing flow of a viscous fluid has been discussed. Conservation laws combined with similarity transformations have been used to formulate the flow mathematically that leads to a highly nonlinear ordinary differential equation. Analytical solution to the resulting differential equation is determined by employing Variation of Parameters Method (VPM). Runge–Kutta order-4 method is also used to solve the same problem for the sake of comparison. It is found that solution using VPM reduces the computational work yet maintains a very high level of accuracy. The influence of different parameters is also discussed and demonstrated graphically.

1. Introduction

Because of its many industrial and practical applications, squeezing flow between parallel plates is an important area of study. These include polymer process industry, compression, injection modeling and the squeezed films in power transmission. The first ever work this regard was done back in 19th century by Stefen [1]. Later Reynolds in 1886 [2] and Archibald [3] in 1956 studied the same problem for elliptic plate and rectangular plates respectively. Since then many researchers have contributed their efforts concerning solution to this problem in different geometries [4–11].

In many physical situations the fluid under consideration is electrically conducting and even a slight presence of magnetic or electromagnetic field can change the behavior of flow. It was therefore essential to discuss the flow under the influence of magnetic field to see how it affects the flow behavior.

Effects of magnetic field on squeezing flow for different geometries have been considered in [12–15] by different researchers. Islam et al. [16] considered it and worked on squeezing flow problem by studying the effects of magnetic field. They used Optimal Homotpic Asymptotic Method (OHAM) to solve the governing equation. However in OHAM we face a lot of laborious work and determination of C_is is not an easy task. So an easier and improved solution is needed. For the fulfillment of this purpose we present this study and employ a technique known as variation of parameters (VPM).

Variation of Parameters Method (VPM) is an analytical technique used to solve different types of problems. It is free

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from round off errors, calculation of the so-called Adomian's polynomials, perturbation, linearization or discretization. It uses only the initial conditions which are easier to be implemented and reduces the computational work while still maintaining a higher level of accuracy. Ma et al. [17,18] employed Variation of Parameters Method (VPM) for solving involved non-homogeneous partial differential. Recently, Mohyud-Din et al. used VPM to solve different problems involving ordinary differential equations, partial differential equations and integral equations [19–26].

In this article, VPM is successfully applied to solve the squeezing flow problem. Comparison of analytical results obtained by this study to the numerical ones is also provided which shows effectiveness of this technique. There is a significant decrease in computational work using this method, yet the results are more accurate.

2. Mathematical formulation

The equations of motion for the flow are given by [12],

$$\nabla \cdot \mathbf{V} = 0, \tag{5}$$

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right] = \nabla\cdot\mathbf{T} - \mathbf{f_B}. \tag{6}$$

where, $\mathbf{V}$ is velocity vector, ρ constant density and T is the Cauchy Stress tensor given by,

$$\begin{aligned}\mathbf{T} &= -p\mathbf{I} + \mathbf{A}_1,\\ \mathbf{A}_1 &= (\nabla\mathbf{V}) + (\nabla\mathbf{V})^T.\end{aligned} \tag{7}$$

Further $\mathbf{f_B}$ is a source term arising due to applied magnetic field, i-e., the so called magnetic or Lorentz force. This force is known to be a function of the imposed magnetic field $\mathbf{B}$, the induced electric field $\mathbf{E}$ and the fluid velocity vector $\mathbf{V}$, that is

$$\mathbf{f_B} = \sigma(\mathbf{E} + \mathbf{V}^*\mathbf{V})^*\mathbf{B}.$$

Consider a steady axisymmetric flow where $\mathbf{V}$ is represented as $\mathbf{V} = [u(r, z, t), 0, w(r, z, t)]$ and the generalized pressure $p^*$and the vorticity $\Omega(r, z)$ are given by,

$$p^* = p + \frac{\rho}{2}|\mathbf{V}|^2, \tag{8}$$

$$\Omega = \frac{\partial w}{\partial r} - \frac{\partial u}{\partial z}. \tag{9}$$

Introducing the stream functions by

$$u(r,z,t) = \frac{1}{r}\frac{\partial\psi}{\partial z},\; w(r,z,t) = -\frac{1}{r}\frac{\partial\psi}{\partial r}. \tag{10}$$

Using Eqs. (10) and (5), continuity equation is identically satisfied. Also, by using Eqs. (7)–(10) in Eq. (6), we have,

$$\frac{\partial p^*}{\partial r} + \frac{\rho}{r}\frac{\partial^2\psi}{\partial t\partial z} - \rho\frac{\partial\psi}{\partial r}\frac{E^2\psi}{r^2} - \frac{\mu}{\rho}\frac{\partial E^2\psi}{\partial z} + \frac{\sigma\mathbf{B}_0^2}{r}\frac{\partial\psi}{\partial z} = 0, \tag{11}$$

$$\frac{\partial p^*}{\partial r} - \frac{\rho}{r}\frac{\partial^2\psi}{\partial t\partial z} - \rho\frac{\partial\psi}{\partial z}\frac{E^2\psi}{r^2} + \frac{\mu}{r}\frac{\partial E^2\psi}{\partial z} = 0. \tag{12}$$

Eliminating pressure from Eqs. (11) and (12), we get the compatibility equation,

$$\rho\left[\frac{1}{r}\frac{\partial E^2\psi}{\partial t} - \frac{\partial(\psi,\frac{E^2\psi}{r^2})}{\partial(r,z)}\right] = \frac{\mu}{r}E^4\psi - \frac{\sigma\mathbf{B}_0^2}{r}\frac{\partial\psi}{\partial z}, \tag{13}$$

where, $E^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} = 0.$

Consider the axisymmetric squeezing flow of an incompressible viscous fluid between two infinite parallel plates. For small values of velocity of the plates, gap 2H changes slowly with the time and flow can be assumed to quasi-steady [27]. Thus, from Eq. (13) we get

$$-\rho\left[\frac{\partial\left(\psi,\frac{E^2\psi}{r^2}\right)}{\partial(r,z)}\right] = \frac{\mu}{r}E^4\psi - \frac{\sigma\mathbf{B}_0^2}{r}\frac{\partial\psi}{\partial z}, \tag{14}$$

with associated auxiliary conditions

$$z = H,\text{ then } u = 0,\; w = -V, \tag{15a}$$

$$z = 0,\text{ then } w = 0,\; \frac{\partial u}{\partial z} = 0. \tag{15b}$$

We can now define stream function as

$$\psi(r,z) = r^2 f(z). \tag{16}$$

Consuming Eq. (16), the compatibility Eq. (14) reduces to

$$f^{i}(z) + 2\frac{\rho}{\mu}f(z)f'''(z) - \frac{\sigma\mathbf{B}_0^2}{\mu r}f'(z) = 0, \tag{17}$$

Subject to the boundary conditions

$$\begin{aligned}&f(0) = 0,\; f''(0) = 0,\\ &f(H) = \frac{V}{2},\; f'(H) = 0.\end{aligned} \tag{18}$$

Eqs. (17) and (18) can be made dimensionless by introducing following the non-dimensional parameters

$$F^* = \frac{f}{V/2},\; z^* = \frac{z}{H},\; \mathrm{Re} = \frac{\rho H}{\mu/V},\; M^2 = \sqrt{\frac{\sigma H\mathbf{B}_0^2}{\mu}}. \tag{19}$$

After omitting the '*' for the sake of simplicity, we may obtain

$$F^{i}(z) + \mathrm{Re}F(z)F'''(z) - M^2F'(z) = 0, \tag{20}$$

with boundary conditions,

$$\begin{aligned}&F(0) = 0,\; F'(0) = 0,\\ &F(1) = 1,\; F'(1) = 0.\end{aligned} \tag{21}$$

3. Solution procedure for VPM

Following the procedure proposed for variation of parameters method [19–26] to solve Eq. (9) with the associated boundary conditions (10), we have

$$\begin{aligned}F_{n+1}(z) = {}& A_1 + A_2 z + A_3\frac{z^2}{2} + A_4\frac{z^3}{6}\\ &- \int_0^z\left(\frac{z^3}{3!} - \frac{z^2 s}{2!} + \frac{zs^2}{2!} - \frac{s^3}{3!}\right)(\mathrm{Re}F_n(s)F_n'''(s)\\ &- M^2F_n'(s))\,ds,\end{aligned} \tag{22}$$

with $n = 0, 1, 2, \ldots$