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A reliable approach for two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability using sumudu transform

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Abstract In this paper, a reliable approach based on homotopy perturbation method using sumudu transform is proposed to compute an approximate solution of the system of nonlinear differential equations governing the problem of two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. This method is called homotopy perturbation sumudu transform method (HPSTM). The technique finds the solution without any discretization or restrictive assumptions and avoids the round-off errors. The numerical solutions show that the proposed method is very efficient and computationally attractive. It provides more realistic series solutions that converge very rapidly for nonlinear real physical problems.

1. Introduction

The flow of Newtonian and non-Newtonian fluids in a porous surface channel has attracted the interest of many researchers in view of its applications in science and engineering practice, particularly in chemical industries. Examples of these are the cases of boundary layer control, transpiration cooling, and gaseous diffusion [1,2]. Berman [3] was first who found a series solution for the two-dimensional laminar flow of a viscous incompressible fluid in a parallel-walled channel for the case of a very low cross-flow Reynolds number. After his pioneering work, many authors have studied this problem by considering various variations in the problem, e.g., Choi et al. [4] and references cited therein. For the case of a converging or diverging channel with a permeable wall, if the Reynolds number is large and if there is suction or injection at the walls whose magnitude is inversely proportional to the distance along the wall from the origin of the channel, a solution for laminar boundary layer equations can be obtained [5].

Most of the scientific and engineering problems such as two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability and other fluid mechanics problems are inherently nonlinear [6,7]. There exists

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a wide class of the literature dealing with the problems of approximate solutions to nonlinear differential equations with various different methodologies, called perturbation methods. The perturbation methods have some limitations e.g., the approximate solution involves series of small parameters which poses difficulty since majority of nonlinear problems have no small parameters at all. Although appropriate choices of small parameters some time lead to ideal solution, but in most of the cases, unsuitable choices lead to serious effects in the solutions. Therefore, an analytical method is welcome which does not require a small parameter in the equation modeling the phenomenon. The homotopy perturbation method (HPM) was first introduced by Chinese researcher J.H. He in 1998 and was developed by him [8–16]. The HPM was also employed by many researchers to investigate the various physical problems [17–25]. Variational iteration method (VIM) was applied to obtain solution of nonlinear wave propagation in shallow media [26]. He's energy balance method (HEBM) was used to study various physical problems [27–29]. A relationship between three analytical approaches was studied to handle nonlinear problems [30]. In recent years, many authors have paid attention to study the solutions of linear and nonlinear partial differential equations by using various methods combined with the Laplace transform and sumudu transform. Among these are Laplace decomposition method (LDM) [31–34], homotopy perturbation transform method (HPTM) [35–37], and homotopy perturbation sumudu transform method (HPSTM) [38,39]. The HPSTM is a combination of sumudu transform method, HPM, and He's polynomials and is mainly due to Ghorbani [40,41].

The objective of this paper is to present a simple recursive algorithm based on the HPSTM which produces the series solution of the two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. The advantage of this technique is its capability of combining two powerful methods for obtaining exact and approximate analytical solutions for nonlinear equations. The fact that HPSTM solves linear and nonlinear problems without using Adomian's polynomials can be considered as a clear advantage of this technique over this decomposition method. It is worth mentioning that the proposed method is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result; the size reduction amounts to an improvement of the performance of the approach.

2. Sumudu transform

In early 90s, Watugala [42] introduced a new integral transform, named the sumudu transform and applied it to the solution of ordinary differential equation in control engineering problems. The sumudu transform is defined over the set of functions

$$\mathrm{A} = \{f(t) | \exists \mathrm{M}, \tau_1, \tau_2 > 0, |f(t)| < \mathrm{M}e^{|t|/\tau_j}, \text{if } t \in (-1)^j \times [0, \infty)\}$$

by the following formula

$$\bar{f}(u) = \mathrm{S}[f(t)] = \int_0^{\infty} f(ut)e^{-t}dt, u \in (-\tau_1, \tau_2). \quad (1)$$

Some of the properties were established by Weerakoon [43,44]. Further, fundamental properties of this transform were developed by Asiru [45] and Belgacem et al. [46–48]. This transform was applied to the one-dimensional neutron transport equation in [49] by Kadem. In fact, it was shown that there is strong relationship between sumudu and other integral transform, see Kilicman et al. [50]. In particular, the relation between sumudu transform and Laplace transforms was proved in Kilicman and Eltayeb [51]. Next, in Eltayeb et al. [52], the sumudu transform was extended to the distributions, and some of their properties were also studied in Kilicman and Eltayeb [53]. Recently, this transform is applied to solve the system of differential equations, see Kilicman et al. [54]. Note that a very interesting fact about sumudu transform is that the original function and its sumudu transform have the same Taylor coefficients except the factor n, see Zhang [55]. Thus if $f(t) = \sum_{n=0}^{\infty} a_n t^n$ then $\bar{f}(u) = \sum_{n=0}^{\infty} n! a_n u^n$,, see Belgacem and Karaballi [47] and Kilicman et al. [50]. Similarly, the sumudu transform sends combinations, $C(m,n)$, into permutations, $P(m,n)$, and hence, it will be useful in the discrete systems.

3. Basic idea of HPSTM

To illustrate the basic idea of this method, we consider a general nonlinear non-homogenous partial differential equation of the form:

$$LU + RU + NU = g(x), \quad (2)$$

where L is the highest order linear differential operator, R is the linear differential operator of less order than L, N represents the general nonlinear differential operator, and $g(x)$ is the source term. By applying the sumudu transform on both sides of Eq. (2), we get

$$S[U] = u^n \sum_{k=0}^{n-1} \frac{U^{(k)}(0)}{u^{(n-k)}} + u^n S[g(x)] - u^n S[RU + NU] = 0. \quad (3)$$

Now applying the inverse sumudu transform on both sides of Eq. (3), we get

$$U = G(x) - S^{-1}[u^n S[RU + NU]], \quad (4)$$

where $G(x)$ represents the term arising from the source term and the prescribed initial conditions. Now, we apply the HPM

$$U = \sum_{m=0}^{\infty} p^m U_m \quad (5)$$

and the nonlinear term can be decomposed as

$$NU = \sum_{m=0}^{\infty} p^m H_m, \quad (6)$$

for some He's polynomials [41,56] that are given by

$$H_m(U_0, U_1, ..., U_m) = \frac{1}{m!} \frac{\partial^m}{\partial p^m} \left[N\left(\sum_{i=0}^{\infty} p^i U_i \right) \right]_{p=0}, m = 0, 1, 2, 3, \ldots \quad (7)$$

Substituting Eqs. (5) and (6) in Eq. (4), we get

$$\sum_{m=0}^{\infty} p^m U_m = G(x) - p\left(S^{-1}\left[u^n S\left[R\sum_{m=0}^{\infty} p^m U_m + \sum_{m=0}^{\infty} p^m H_m \right] \right] \right), \quad (8)$$