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Topological soliton solutions for some nonlinear evolution equations

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Abstract In this paper, the topological soliton solutions of nonlinear evolution equations are obtained by the solitary wave ansatz method. Under some parameter conditions, exact solitary wave solutions are obtained. Note that it is always useful and desirable to construct exact solutions especially soliton-type (dark, bright, kink, anti-kink, etc.) envelope for the understanding of most nonlinear physical phenomena.

1. Introduction

Nonlinear evolution equations are special classes of the category of partial differential equations (PDEs), which have been studied intensively in past few decades [1]. It is well known that seeking explicit solutions for nonlinear evolution equations, by using different numerous methods, plays a major role in mathematical physics and becomes one of the most exciting and extremely active areas of research investigation for mathematicians and physicists. In fact, when we want to understand the physical mechanism of phenomena in nature, described by nonlinear PDEs, exact solution for the nonlinear PDEs has to be explored. In particular, the soliton solutions play an important role in the study of the models arising from various natural phenomena and scientific fields; for instance, the wave phenomena observed in fluid mechanics, elastic media, optical fibers, nuclear physics, high-energy physics, plasma physics, biology, solid-state physics, chemical kinematics, chemical physics and geochemistry, etc. Most famous models of such equations admitting solitons are, for example, the nonlinear Schrödinger equations, the Korteweg–de Vries equations, the Kadomtsev–Petviashvili equations, the Boussinesq equations, and the Zakharov–Kuznetsov equations. A large number of such equations have been studied in these contexts, and numerous analytic and computational effective techniques have been proposed to investigate these types of equations.

Some of these methods that have been recently developed are the $\left(\frac{G'}{G}\right)$-expansion method [2,3], exponential function method [4,5], Fan's F-expansion method [6,7], the tanh–sech method [8–10], extended tanh method [11–13], sine–cosine method [14–16], homogeneous balance method [17,18], first integral method [19–21], simplest equation method [22,23], ansatz method [24–27] and many others. There are various

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advantages and disadvantages of these modern methods of integrability that includes these methods. Although a closed form soliton solution can be obtained by these techniques, the disadvantage of these methods is that these techniques cannot compute the conserved quantities of nonlinear evolution equations nor it can lay down an expression of the soliton radiation. Nevertheless, the fact that soliton solutions can be obtained is itself a big blessing [28–30].

In this paper, one such method of integration will be used to carry out the integration for some nonlinear evolution equations (NEEs) to obtain the topological soliton solutions.

2. Ansatz method

In this section, the ansatz method will be used to obtain the Calogero and Ramani equations of solutions. The search is going to be for a topological 1-soliton solution, which is also known as a kink solution or a shock wave solution. This will be demonstrated in the following two subsections. For both equations, arbitrary constant coefficients will be considered.

2.1. (2 + 1)-Dimensional breaking soliton (Calogero) equation

Consider the following nonlinear (2 + 1)-dimensional breaking soliton (Calogero) equation is given

$$u_{xxxy} - 2u_y u_{xx} - 4u_x u_{xy} + u_{xt} = 0 \tag{1}$$

which was first introduced by Calogero et al. [31]. Mei and Zhang obtained some soliton-like solutions and periodic solutions by using the projective Riccati equation expansion method [32]. Geng and Cao are obtained some N-soliton solutions and algebro-geometric solutions [33].

The topological soliton solution: The topological solitons that are also known as optical solitons are also supported by the NEEs [34–39]. In this case, as it will be seen that for the case of generalized NEEs, the topological solitons will exist under certain restrictions. The subject of topological solitons will be studied for the case of power law nonlinearity only. Thus, the equation of study in this section will be (1). Let us begin the analysis by assuming an ansatz solution of the from [40–45]:

$$u(x, y, t) = \lambda \tanh^p \tau \tag{2}$$

and

$$\tau = \eta_1 x + \eta_2 y - vt \tag{3}$$

where the parameters λ, η_i $(i = 1, 2)$ are the free parameters and is the velocity of the soliton. The value of the unknown exponent p will be determined during the course of derivation of the soliton solution of (1)

From Eqs. (2) and (3), we find $u_x, u_y, u_{xt}, u_{xx}, u_{xy}, u_y u_{xx}, u_x u_{xy}, u_{xxxy}$ and substituting these equations into Eq. (1), we obtain

$$\begin{aligned} &p\eta_1^3\eta_2\lambda\{(p-1)(p-2)(p-3)\tanh^{p-4}\tau - 4(p-1)(p^2-2p \\ &\quad +2)\tanh^{p-2}\tau + 2p(3p^2+5)\tanh^p\tau\} - p\eta_1^3\eta_2\lambda\{4(p+1)(p^2 \\ &\quad +2p+2)\tanh^{p+2}\tau - (p+1)(p+2)(p+3)\tanh^{p+4}\tau\} \\ &\quad - 6\lambda^2p^2\eta_1^2\eta_2\{(p-1)\tanh^{2p-3}\tau - (3p-1)\tanh^{2p-1}\tau + (3p \\ &\quad +1)\tanh^{2p+1}\tau - (p+1)\tanh^{2p+3}\tau\} - p\lambda v\eta_1\{(p \\ &\quad +1)\tanh^{p+2}\tau - 2p\tanh^p\tau + (p-1)\tanh^{p-2}\tau\} \\ &\quad = 0 \end{aligned} \tag{4}$$

Now equating the highest exponents of $\tanh^{2p+1}\tau$ and $\tanh^{p+2}\tau$ terms in Eq. (4) gives

$$2p + 1 = p + 2 \tag{5}$$

which yields

$$p = 1 \tag{6}$$

It should be noted that the same value of p is yielded when the exponents pair $2p + 3$ and $p + 4$; $2p - 3$ and $p - 2$; and $2p - 1$ and p are equated with each other, respectively.

$$p(p+1)(p+2)(p+3)\eta_1^3\eta_2\lambda + 6\lambda^2p^2(p+1)\eta_1^2\eta_2 = 0 \tag{7}$$

$$-4p(p+1)(p^2+2p+2)\eta_1^3\eta_2\lambda - 6\lambda^2p^2(3p+1)\eta_1^2\eta_2 - p(p+1)\lambda v\eta_1 = 0 \tag{8}$$

$$2p^2(3p^2+5)\eta_1^3\eta_2\lambda + 6\lambda^2p^2(3p-1)\eta_1^2\eta_2 + 2p^2\lambda v\eta_1 = 0 \tag{9}$$

If we put $p = 1$ in (7)–(9), the system reduces to

$$24\eta_1^3\eta_2\lambda + 12\lambda^2\eta_1^2\eta_2 = 0 \tag{10}$$

$$-40\eta_1^3\eta_2\lambda - 24\lambda^2\eta_1^2\eta_2 - 2\lambda v\eta_1 = 0 \tag{11}$$

$$16\eta_1^3\eta_2\lambda + 12\lambda^2\eta_1^2\eta_2 + 2\lambda v\eta_1 = 0 \tag{12}$$

Solving the above equations yields

$$\lambda = -2\eta_1 \tag{13}$$

$$v = 4\eta_1^2\eta_2 \tag{14}$$

Hence, finally, the topological 1-soliton solution to (1) is respectively given by

$$u(x, y, t) = -2\eta_1 \tanh\left(\eta_1 x + \eta_2 y - 4\eta_1^2\eta_2 t\right) \tag{15}$$

and where the parameter λ is given by (13), the velocity of the solitons is given in (14).

2.2. The sixth-order Ramani equation

We consider the sixth-order Ramani equation or the KdV6 equation is given [46]

$$\begin{aligned} &.u_{xxxxxx} + 15u_x u_{xxxx} + 15u_{xx}u_{xxx} + 45u_x^2 u_{xx} \\ &\quad - 5(u_{xxxt} + 3u_x u_{xt} + 3u_t u_{xx}) - 5u_{tt} = 0 \end{aligned} \tag{16}$$

In [47], it was indicated that there are four distinct cases for Eq. (16) to pass the Painlevé test and was examined for complete integrability by using the Bäcklund transformation and Lax pairs. Now, topological soliton solution of this equation will be obtained. The Hirota's bilinear method is used to determine the two distinct structures of solutions by Wazwaz and Triki in [48].

The topological soliton solution: In this section, we are interested in finding the topological soliton solution, as defined in, [49–54] for the considered sixth-order Ramani Eq. (16).

$$u(x, t) = \lambda \tanh^p \tau \tag{17}$$

and

$$\tau = \eta(x - vt) \tag{18}$$

Here, λ, η will be determined free parameters and is the soliton velocity. The unknown index p will be determined in terms of derivation of the exact solution. From Eqs. (17) and (18), it is possible to obtain u_{xxxxxx}, $u_x u_{xxxx}$, $u_t u_{xx}$, $u_{xx}u_{xxx}$, u_{xxxt}, $u_x u_{xt}$, $u_x^2 u_{xx}$, u_{tt} these terms and substituting these equations into Eq. (16), we obtain