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Eigenvalue approach to fractional order generalized magneto-thermoelastic medium subjected to moving heat source

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ABSTRACT

In the present work, we consider the problem of fractional order thermoelastic interaction in a material placed in a magnetic field and subjected to a moving plane of heat source. The basic equations have been written in the form of a vector–matrix differential equation in the Laplace transform domain, which is then solved by an eigenvalue approach. The inverse Laplace transforms are computed numerically and some comparisons have been shown in figures to estimate the effect of each of the fractional order, heat source velocity, time and the magnetic field and parameters.

1. Introduction

The generalized theories of thermoelasticity, which admit the finite speed of thermal signal, have been the center of interest of active research during last three decades. Biot [1] introduced the theory of coupled thermoelasticity to overcome the first shortcoming in the classical uncoupled theory of thermoelasticity where it predicts two phenomena not compatible with physical observations. The theory of couple thermoelasticity was extended by Lord and Shulman [2] and Green and Lindsay [3] by including the thermal relaxation time in constitutive relations. The theory was extended for anisotropic body by Dhaliwal and Sherief [4]. The counterparts of our problem in the contexts of the thermoelasticity theories have been considered by using analytical and numerical methods [5–17].

Fractional calculus has been used successfully to modify many existing models of physical processes. One can state that the whole theory of fractional derivatives and integrals was established in the second half of the nineteenth century. Various definitions and approaches of fractional derivatives have become the main purpose of many studies. Youssef [18,19] established the fractional order generalized thermoelasticity of both weak and strong heat conductivity in the context of generalized thermoelasticity were considered. Ezzat and Karamany [20–22] established a new model of fractional heat equation based on a Taylor expansion of time-fractional order. In addition, Sherief et al. [23] established a new model by using the form of the heat conduction law. Kumar et al. [24] studied the plane deformation due to thermal source in fractional order thermoelastic media.

In the present paper we have applied the technique of eigenvalue approach developed in [25] to solve a problem of fractional order theory of generalized magneto-thermoelastic subjected to a moving plane of heat source. The inversion of Laplace transform have been carried out numerically by applying a method of numerical inversion of Laplace transform based on Stehfest technique [26]. The variables in physical space–time domain are represented graphically.

2. Basic equation and formulation of the problem

Following Ezzat and El-Karamany [20], the basic equations of fraction order theory of magneto-thermoelastic medium of perfect conductivity permeated by an initial magnetic field H are considered as

The first set of equations constitutes the equations of electrodynamics of slowly moving bodies,

$$\mathrm{curl}\,\mathbf{h} = \mathbf{J} + \varepsilon_o \frac{\partial \mathbf{E}}{\partial t},\ \mathrm{curl}\,\mathbf{E} = -\mu_o \frac{\partial \mathbf{h}}{\partial t},\ \mathbf{E} = -\mu_o\left(\frac{\partial u}{\partial t} \wedge \mathbf{H}\right),\ \mathrm{div}\,\mathbf{h} = 0. \tag{1}$$

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The equations of motion

$$\sigma_{ji,j} + \mu_o (J \wedge H)_i = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{2}$$

The equation of heat conduction with fractional time derivatives

$$KT_{ii} = \left(1 + \frac{\tau_o^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho c_e \dot{T} + \gamma T_0 \dot{e} - Q\right), \; 0 < \alpha \le 1. \tag{3}$$

The constitutive equations are given by

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma (T - T_0)]\delta_{ij}, \tag{4}$$

where **h** is induced magnetic field; **E** is the induced electric field; **J** is the current density vector, **B** is the magnetic inductance vector and μ_o is the magnetic permeability; ε_o is the electric permittivity; λ, μ are Lame's constants; ρ is the density of the medium; c_e is the specific heat at constant strain; $\gamma = (2\lambda + 3\mu)\alpha_t$ and α_t is the coefficient of linear thermal expansion; t is the time; T is the temperature; T_0 is the reference temperature; K is the thermal conductivity; τ_o is the thermal relaxation time;δ_{ij} is the Kronecker symbol; σ_{ij} are the components of stress tensor; u_i are the components of displacement vector and Q is the moving heat source.

Let us consider a homogeneous isotropic thermoelastic solid at a uniform reference temperature T_0 of a perfect electrically conductivity permeated by an initial magnetic field $H_0 = (0, H_0, 0)$, occupying the region $x \ge 0$ where the x-axis is taken perpendicular to the bounding plane of the half-space pointing inwards. It assumed that the state of the medium depends only on x and the time variable t, so that the displacement vector **u** and temperatures field T can be expressed in the following form:

$$\mathbf{u}_x = \mathbf{u}(x, t), \; \mathbf{u}_y = 0, \; \mathbf{u}_z = 0, \; T = T(x, t). \tag{5}$$

Then the Eqs. (2)–(4) take the following form

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} + \mu_o H_0^2 \frac{\partial^2 u}{\partial x^2} - \varepsilon_o \mu_o^2 H_0^2 \frac{\partial^2 u}{\partial t^2} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{6}$$

$$K \frac{\partial^2 T}{\partial x^2} = \left(1 + \frac{\tau_o^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho c_e \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial^2 u}{\partial t \partial x} - Q\right), \tag{7}$$

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma (T - T_0). \tag{8}$$

Now, we introduce the following non-dimensional variables

$$(x', u') = c_1 \eta (x, u),$$
$$T' = \frac{\gamma (T - T_0)}{\lambda + 2\mu},$$
$$(t', \tau_o') = c_1^2 \eta (t, \tau_o),$$
$$\sigma_{xx}' = \frac{\sigma_{xx}}{\lambda + 2\mu},$$
$$h' = \frac{h}{H_0},$$
$$E' = \frac{E}{\mu_o H_0 c_1},$$
$$Q' = \frac{\gamma Q}{K \rho c^4 \eta^2},$$

where $c_1^2 = (\lambda + 2\mu/\rho)$ and $\eta = (\rho c_e/K)$.

Upon introducing in Eqs. (6)–(8), and after suppressing the primes, we obtain

$$\beta_1^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial T}{\partial x} = \beta_2^2 \frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$\frac{\partial^2 T}{\partial x^2} = \left(1 + \frac{\tau_o^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\frac{\partial T}{\partial t} + \varepsilon \frac{\partial^2 u}{\partial t \partial x} - Q\right), \tag{10}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} - T, \tag{11}$$

where $\beta_1^2 = 1 + (R_H/c_1)^2$, $\beta_2^2 = 1 + (R_H/c)^2$, $R_H^2 = (\mu_o H_0^2/\rho)$, $c^2 = (1/\varepsilon_o \mu_o)$, $\varepsilon = \left(T_0 \gamma^2/\rho_o c_e (\lambda + 2\mu)\right)$.

We consider that the medium is subjected to a moving heat source of constant strength releasing its energy continuously while moving along the positive direction of the x-axis with a constant velocity v. This moving heat source is assumed to be the following non-dimensional form

$$Q = Q_o \delta (x - vt), \tag{12}$$

where Q_o is constant and δ is the delta function. We assume that the medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have

$$u(x, 0) = \frac{\partial u(x, 0)}{\partial t} = 0, \; T(x, 0) = \frac{\partial T(x, 0)}{\partial t} = 0. \tag{13}$$

The boundary conditions can be written as

$$u(0, t) = 0, \; \frac{\partial T(0, t)}{\partial x} = 0. \tag{14}$$

Applying the Laplace transform define by the formula

$$\bar{f}(s) = L[f(t)] = \int_0^\infty f(t) e^{-st} dt, \; \mathrm{Re}(s) > 0. \tag{15}$$

Hence, we obtain the following system of differential equations

$$\beta_1^2 \frac{d^2 \bar{u}}{dx^2} - \frac{d\bar{T}}{dx} = \beta_2^2 s^2 \bar{u}, \tag{16}$$

$$\frac{d^2 \bar{T}}{dx^2} = \left(1 + \frac{s^\alpha \tau_o^\alpha}{\alpha!}\right)\left(s\bar{T} + s\varepsilon \frac{d\bar{u}}{dx} - \frac{Q_o}{v} e^{-sx/v}\right), \tag{17}$$

$$\bar{\sigma}_{xx} = \frac{d\bar{u}}{dx} - \bar{T}, \tag{18}$$

$$\bar{u}(0, s) = 0, \; \frac{d\bar{T}(0, s)}{dx} = 0. \tag{19}$$

Eqs. (16) and (17) can be written in a vector–matrix differential equation as follows [25]:

$$\frac{d\vec{V}}{dx} = B\vec{V} + \vec{f}, \tag{20}$$

where $\vec{V} = \left[\bar{u} \;\; \bar{T} \;\; \frac{d\bar{u}}{dx} \;\; \frac{d\bar{T}}{dx}\right]^T$, $B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ b_{31} & 0 & 0 & b_{34} \\ 0 & b_{42} & b_{43} & 0 \end{bmatrix}$ and

$$\vec{f} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_4 e^{-sx/v} \end{bmatrix},$$

with $b_{31} = \frac{\beta_2^2 s^2}{\beta_1^2}$, $b_{34} = \frac{1}{\beta_1^2}$, $b_{42} = s + \frac{s^{\alpha+1} \tau_o^\alpha}{\alpha!}$, $b_{43} = \varepsilon b_{42}$, $f_4 = -\frac{b_{42} Q_o}{sv}$.