Journal of Magnetism and Magnetic Materials 377 (2015) 496–501

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

The complex initial reluctivity, permeability and susceptibility spectra of magnetic materials

N.C. Hamilton

North View, Chawston Lane, Chawston, Bedford MK44 3BH, UK

ARTICLE INFO

Article history:
Received 24 February 2014
Received in revised form
13 September 2014
Accepted 17 October 2014
Available online 22 October 2014

Keywords:
Complex permeability spectrum
Reluctivity
Magnetic susceptibility
Soft ferrite

ABSTRACT

The HF complex permeability spectrum of a magnetic material is deduced from the measured impedance spectrum, which is then normalized to a series permeability spectrum. However, this series permeability spectrum has previously been shown to correspond to a parallel magnetic circuit, which is not appropriate. Some of the implications of this truth are examined. This electric/magnetic duality has frustrated efforts to interpret the shape of the complex magnetic permeability spectra of materials, and has hindered the application of impedance spectroscopy to magnetic materials. In the presence of magnetic loss, the relationship between the relative magnetic permeability and the magnetic susceptibility is called into question. The use of reluctivity spectra for expressing magnetic material properties is advocated. The relative loss factor, $\tan\delta_m/\mu_i$ is shown to be an approximation for the imaginary part of the reluctivity. A single relaxation model for the initial reluctivity spectra of magnetic materials is presented, and its principles are applied to measurements of a high permeability ferrite. The results are presented as contour plots of the spectra as a function of temperature.

1. Introduction

1947 was very significant in the history of ferrite and of magnetism in general. Snoek's 1947 book [1] described the preparation of MnZn and NiZn ferrites; these are still the chemical basis of most of the soft ferrites in use today. Snoek's 1947 letter [2] proposed that the complex series susceptibility spectrum of ferrite was due to gyro-magnetic resonance. Also in 1947, Macfadyen [3] gave a detailed description of the use of permeability and reluctivity as complex quantities.

In the earlier part of the century, it had become firm practice to use permeability rather than reluctivity, and as long as these were viewed as scalar quantities, this caused no difficulties. Permeability was probably chosen because the values were of a convenient size when using the CGS system of units, and because it seems more natural for a magnetic field strength to cause a flux density. But by 1950, the use of the complex series permeability spectra $\boldsymbol{\mu}_{r,s}(f)=\mu'(f)\text{-}j\mu''(f)$ in SI units for small-signal material characterization was becoming standard practice. It remains so today, in spite of successive observations [4–7] that the complex permeability spectra of ferrites at HF are simpler when expressed in parallel terms, rather than in series terms.

Snelling [8] and Wijn [9] state that it was Snoek and Six who established an alternative measure for characterizing materials: the relative loss factor $\tan\delta_m/\mu_i$ (or its reciprocal, $\mu_i Q_m$, probably due to Owens [10]), where μ_i is the initial permeability, a scalar value measured when both frequency and flux density are very small. The relative loss factor is widely used by manufacturers of soft ferrites, especially in Northern America.

Since 1970s, work has progressed from material characterization to the interpretation and simulation of the permeability spectra. In the author's view, the exclusive use of complex series permeability is hindering progress on the subject. This article reviews the basis of permeability spectra with the intention of encouraging the use of complex reluctivity or parallel permeability.

2. Theory

2.1. Reluctivity and permeability

Consider the magnetic circuit in a ring core. The components of energy storage and of energy loss in the core are homogenously distributed throughout the material, so all the flux change in the core is subject to both energy storage and dissipation. If the core's distributed energy storage were to be lumped in one sector of the core, and the magnetic losses in the other sector, then the lumped element small-signal magnetic circuit must consist of these two material types in series.

Macfadyen [3] observed "It is generally best to work with permeability when there are magnetic paths in parallel and with

E-mail address: nchamilton@theiet.org

http://dx.doi.org/10.1016/j.jmmm.2014.10.061
0304-8853/

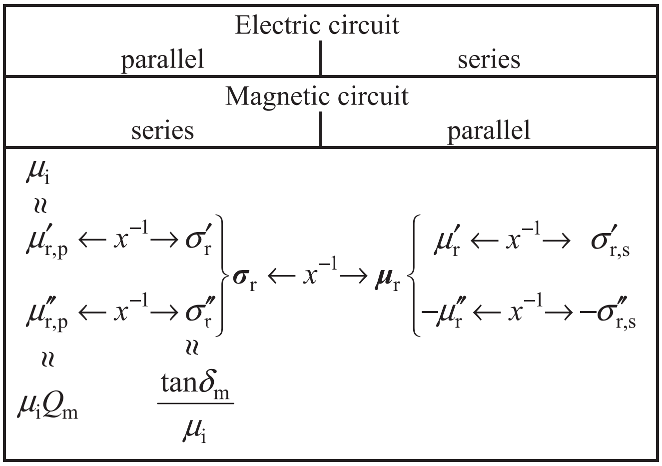

Fig. 1. Reciprocals (x^{-1}) relate the various forms of complex relative permeability $\boldsymbol{\mu}_r$ and reluctivity $\boldsymbol{\sigma}_r$ and their commonly used approximations. The series components of reluctivity $\sigma_{r,s}$ are shown for completeness, but are not used.

reluctivity when the magnetic paths are in series…". Macfadyen then showed that the complex reluctivity gives a parallel equivalent electrical circuit.

This view was amplified by Cherry [11], who showed that the electric and magnetic circuits were duals of each other. Cherry then showed explicit equivalent circuits of a lossy magnetic material as a series magnetic circuit of real and imaginary parts of reluctance, modelled by an electrical circuit consisting of an admittance: an inductor and a resistor connected in parallel. Fig. 1 shows the relationships between the series and parallel forms of both permeability and reluctance, where the sub-scripts denoting series and parallel forms are based on IEC 62044-2 2005.

Although characterization of a material's series magnetic energy storage and magnetic loss components is best expressed in terms of complex reluctance, Fig. 1 shows that the same data may be expressed as a so-called parallel permeability, the components of which in fact represent a series magnetic circuit.

It is regrettable that common usage differentiates between the series and parallel forms of these magnetic quantities in terms of the series and parallel dual electric circuit. As Schlicke observed [12] of basic discontinuous material structures: "What seems magnetically a series arrangement is electrically a parallel circuit and vice versa." and that the equivalent electric circuit is "… contrary to what the eye seems to see so clearly in the magnetic structure".

From Fig. 1, it can be deduced that the ratios between the real and imaginary parts of the permeability and reluctivity are constant, so:

$$\frac{\mu''_{r,p}}{\mu'_{r,p}} = \frac{\sigma'_r}{\sigma''_r} = \frac{\mu'_r}{-\mu''_r} = \frac{-\sigma''_{r,s}}{\sigma'_{r,s}} = \frac{1}{\tan\delta_m} = Q_m \tag{1}$$

where Q_m is the material's magnetic quality factor, and $\tan\delta_m$ is the loss factor.

Now, as Macfadyen observed [3]: "Reluctivity possesses the advantage that (σ'_r and hence $\mu'_{r,p}$) is constant over a wide range of frequency…". This is confirmed by examination of the series and parallel permeability spectra of the 22 ferrite grades presented by Snelling [13]. See also the measurements presented in Fig. 5.

It follows that the initial permeability $\mu_i \approx \mu'_{r,p}$, and this is shown in Fig. 1. From this, and the relationships given in Eq. (1), the approximations for the relative loss factor $\tan\delta_m/\mu_i$ and its reciprocal $\mu_i Q_m$ follow. These two factors are approximations for the loss term of the series magnetic circuit, but they conceal this, the origin of their usefulness.

2.2. Susceptibility and permeability

The magneto-static relationship between these quantities as dimensionless scalars is, by definition, $\mu_r = \chi + 1$, where χ is the (rationalized magnetic volume) susceptibility. Assume the simplest susceptibility spectrum, a single relaxation given by a Debye type equation [14] for the complex magnetic susceptibility $\boldsymbol{\chi}(\omega)$:

$$\chi(\omega) = \frac{\chi_0}{1 + j\omega\tau} \tag{2}$$

where χ_0 is the static susceptibility, ω is the angular frequency, τ is the time constant of relaxation and j is the complex operator. There are two possible methods of proceeding [15].

2.2.1. Series susceptibility conversion

Add 1 to the series real component of $\boldsymbol{\chi}(\omega)$, thereby making the assumption that $\boldsymbol{\mu}_r(\omega) = \boldsymbol{\chi}(\omega) + 1$. Resolve $\boldsymbol{\chi}(\omega)$ into real and imaginary parts:

$$\chi'(\omega) = \frac{\chi_0}{1 + (\omega\tau)^2} \text{ and } \chi''(\omega) = \frac{-\chi_0\omega\tau}{1 + (\omega\tau)^2} \tag{3a and 3b}$$

and add 1 to the real part, giving:

$$\mu_r'(\omega) = 1 + \frac{\chi_0}{1 + (\omega\tau)^2} \tag{4a}$$

and

$$\mu_r''(\omega) = \frac{-\chi_0\omega\tau}{1 + (\omega\tau)^2} \tag{4b}$$

Unfortunately, Eqs. (4a) and (4b) do not easily convert to an expression for complex reluctivity. Fig. 2 shows a graph of the 'series' permeability spectra resulting from values of χ_0 and τ appropriate to a ferrite for use in HF/VHF inductors.

At low frequencies, Q_m falls as the frequency increases: this is expected. However, the frequency axis of Fig. 2 is extended to show that, above 800 MHz, Q_m starts to rise again. This is not observed in practice, and is an artefact of the series conversion of susceptibility to permeability, which is unlikely to be correct.

2.2.2. Parallel susceptibility conversion

Add 1 to the **parallel** real component of $\boldsymbol{\chi}(\omega)$. Using the relations given in Fig. 1, convert series susceptibility to its reciprocal. (Magnetic immunity has been suggested [16] as a name for

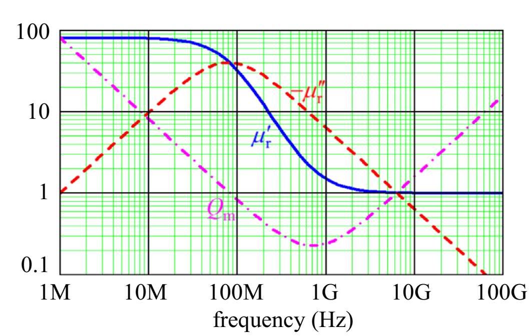

Fig. 2. Series relative permeability spectrum real (solid) and imaginary (dashed) parts modelled for $\chi_0 = 79$ and $\tau = 2$ ns, if $\boldsymbol{\mu}_r(\omega) = \boldsymbol{\chi}(\omega) + 1$. Also shown (dot dashed) is Q_m, which is unrealistic above 800 MHz.