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An analytical investigation on unsteady motion of vertically falling spherical particles in non-Newtonian fluid by Collocation Method

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Abstract An analytical investigation is applied for unsteady motion of a rigid spherical particle in a quiescent shear-thinning power-law fluid. The results were compared with those obtained from Collocation Method (CM) and the established Numerical Method (Fourth order Runge–Kutta) scheme. It was shown that CM gave accurate results. Collocation Method (CM) and Numerical Method are used to solve the present problem. We obtained that the CM which was used to solve such nonlinear differential equation with fractional power is simpler and more accurate than series method such as HPM which was used in some previous works by others but the new method named Akbari-Ganji's Method (AGM) is an accurate and simple method which is slower than CM for solving such problems. The terminal settling velocity—that is the velocity at which the net forces on a falling particle eliminate—for three different spherical particles (made of plastic, glass and steel) and three flow behavior index n, in three sets of power-law non-Newtonian fluids was investigated, based on polynomial solution (CM). Analytical results obtained indicated that the time of reaching the terminal velocity in a falling procedure is significantly increased with growing of the particle size that validated with Numerical Method. Further, with approaching flow behavior to Newtonian behavior from shear-thinning properties of flow ($n \rightarrow 1$), the transient time to achieving the terminal settling velocity is decreased.
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1. Introduction

An important natural phenomenon that occurs in many industrial processes is the sedimentation and falling of solid particles in gases and liquids. Primarily, sedimentation results from a tendency of suspended particles in fluids to settle and come to rest, due to the forces acting on them through the fluid [1]. Common examples include separation of liquid–solid mixtures, sprays and atomization, sediment transportation and

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Nomenclature

m	particle mass
u	particle velocity
g	gravity acceleration
D	particle diameter
C_D	drag coefficient
t	time
Re	Reynolds number
n	flow behavior index
V_0	velocity at $t = 0$
k	consistency coefficient
$X(n)$	deviation factor
j	number of polynomial statements
u_m	terminal settling velocity
t_m	time of terminal settling velocity
CM	Collocation Method
HPM	Homotopy Perturbation Method
HAM	Homotopy Analysis Method
VIM	Variational Iteration Method
NuM	Numerical Method

Greek symbols

ρ	fluid density
ρ_s	particle density

deposition in pipe lines [2,3], alluvial channels [4,5], and chemical and powder processing.

In many processes it is often essential to obtain the route of particles that accelerates in the fluid region for designing or improving the processes. The majority of previous studies have considered the steady-state conditions and where the particles achieved their terminal velocity. Also, several works have been done to study the unsteady motion of particles in Newtonian fluids [6–13] due to its applications in classification, centrifugal collection and separation (some of the unit operations which require the trajectories of particles accelerating in fluid). Further, the distance required to reach the terminal velocity is necessary for viscosity measurements of fluid with the falling ball experiment.

Along with the same proposition, many researchers realized the physical significance of some analytical methods such as the Homotopy Perturbation Method (HPM) [14], Variational Iteration Method (VIM) [15,16] and Homotopy Analysis Method (HAM) [17] and its compatibility with the physical problems as the unsteady motion of spherical particles in Newtonian fluids. Hatami and Ganji introduced the equation of the motion for variable-mass particle for the first time and solved by Padé approximation of Differential Transformation Method (DTM-Padé) and numerical Runge–Kutta method [18].

These methods were originally proposed by He [19,20] to achieve the series solution of strongly nonlinear differential equations. Jalaal et al. [21] used HPM to study the unsteady motion of a spherical particle falling in a Newtonian fluid for a range of Reynolds number to obtain a solution for nonlinear equations of a falling spherical with drag coefficient. Then, Jalaal et al. [22] used a series-based method called Homotopy Analysis Method (HAM) in order to solve nonlinear particle equation of motion whose results are very accurate and reliable. Meanwhile, an unsteady rolling motion of spheres in inclined tubes filled with incompressible Newtonian fluids was conducted by Jalaal et al. [22]. Later, Hamidi et al. [23] applied the HPM-Padé to solve the coupled equations of a spherical solid particle's motion in Couette flow. Hatami et al. solved coupled equations of particle's motion in Couette fluid flow by Multi-step Differential Transformation Method (Ms-DTM) considering the rotation and shear effects on lift force and neglecting gravity [24]. Hatami and Ganji investigated coupled equations of the motion of a particle in a fluid forced vortex the differential transformation method (DTM) with the Padé approximation and the differential quadrature method (DQM) [25]. Hatami and Ganji introduced the equation of a particle's motion on a rotating parabolic surface through Lagrange equations and solved by Multi-step Differential Transformation Method (Ms-DTM) [26].

Majority of the above mentioned studies have described the motion of solid particles in Newtonian suspensions only, however, many slurries and concentrated suspensions, which are treated in the materials processing industry, behave as non-Newtonian liquids and proper consideration has to be made [27–32]. The numerical solution of Bagchi and Chhabra [33] is one of the studies in this field. They reported the distance traveled by accelerating spherical particles in downward vertical motion of particles in power law liquids. Malvandi et al. [34] have studied analytically with HPM and VIM scheme on present problem and their results had very good agreement with the older researches. Therefore, Collocation Method [35,36] was used to find efficient, reliable and precise polynomial solutions. In order to consider the non-Newtonian fluid flow, the power-law model was employed. Furthermore, the terminal settling velocity for three rigid spherical particles namely plastic, glass and steel, vertically falling in the quiescent power-law fluids was determined. In terms of obtaining the best accuracy of the analytical results, a comparison was made by a numerical solution via forth order Runge–Kutta Numerical Method.

2. Problem formulations

In Fig. 1 it is shown the consideration on one-dimensional accelerated motion of a rigid spherical particle vertically falling to an infinite extent of a power-law shear-thinning fluid. The forces acting on a falling body are usually gravity, buoyancy, inertia, Basset history force, virtual mass and drag force. From the Lagrangian viewpoint, the dynamic of particles submerged in a fluid could be obtained by integrating the forces balanced on them. According to the studies of Renganathan et al. [37] and Bagchi and Chhabra [33], the Basset force can be assumed to be negligible when the density of the spherical particle is much larger than that of the liquid. Under this condition, the equation of motion describing the falling motion of the particle can be written as [22],

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