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Role of thermal diffusion on double-diffusive natural convection in a vertical annular porous medium

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Abstract This paper derives analytical solutions of fully developed natural convection heat and mass transfer in a vertical annular non-Darcy porous medium. This investigation extends the work of Cheng (2006) to a situation where Soret effect is present and the flow is both aided and opposed. The influence of the controlling parameters on the flow characteristics has been seen to be higher for double diffusion in the absence of Soret effect than in the presence of Soret effect. It is found that the presence of Soret effect reduces the dependence of the volume flow rate, the total heat rate and the total species rate on the inner radius-gap ratio and on the modified Darcy number.

1. Introduction

The natural convection of binary fluids flow in porous media has attracted great research interest during the past few decades. While a good number of works have made significant contributions for the development of the theory, an equally good number of works have been devoted to the numerous industrial, natural and geophysical applications. Double-diffusive convective flows in a differentially heated vertical annulus have been intensively studied in relation to applications such as oxidation of surface materials, cleaning and dying operations, fluid storage components and energy storage in solar ponds [1]. Consideration of two kinds of problems concerning the convection of a binary mixture filling a porous layer is in the literature. The first kind of problem considers flows induced by the buoyancy forces resulting from the imposition of both thermal and solute boundary conditions on the layer. The second kind of problem considers thermal convection in a binary fluid driven by Soret-effects. For this situation, the species gradients are not due to the imposition of solute boundary conditions. Rather, they result from the imposition of a temperature gradient in an otherwise uniform-concentration mixture. This phenomenon has many applications in geophysics, oil reservoirs, and ground water. Bahloul et al. [2] gave the reviews of previous works done in this direction. A very recent comprehensive overview of double-diffusive convection in saturated porous media, its relevance in the understanding of many

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Nomenclature

a	integer number as $a = 0,\ 1$
C	concentration
D	mass diffusivity
D^*	thermal diffusion coefficient
Da	modified Darcy number
g	gravitational acceleration
Gr	Grashof number
I_0	zero-order modified Bessel function of the first kind
K_0	zero-order modified Bessel function of the second kind
I_1	first-order modified Bessel function of the first kind
K_1	first-order modified Bessel function of the second kind
K	permeability
N	buoyancy ratio
Nu	Nusselt number
Q	volume flow rate
r'	radial coordinate
R	dimensionless radial coordinate
R_c	wall concentration ratio
R_t	wall temperature ratio
T	temperature
u'	velocity component along axis of the annulus
U	dimensionless velocity component along axis of the annulus

Greek symbols

β_c	coefficient of concentration expansion
β_t	coefficient of thermal expansion
ε	porosity
θ	dimensionless temperature
λ	inner radius-gap ratio
ν	kinematics' viscosity
ϕ	dimensionless concentration

Subscripts

0	condition at the inlet
1	condition at the outer surface of the inner cylinder
2	condition at the inner surface of the outer cylinder

natural systems and its wide variety of engineering applications is well documented in the literature [3–7].

Boutana et al. [8] investigated the Soret effect and double-diffusive natural convection in a rectangular porous medium filled with a binary fluid both analytically and numerically. They reported that the range of buoyancy ratios for the existence of multiple solutions depends on the type of convection induced by the solute gradients, i.e. on the constant a. In particular, it was demonstrated that for opposing flows, near the buoyancy ratio greater than -1, multiple solutions are possible for a range of R_t which depends on the other governing parameters. Nithyadevi and Yang [9] considered the effect of double-diffusive natural convection of water in a partially heated enclosure with Soret and Dufour coefficients numerically. They concluded that the fluid particle moves with lesser velocity and high mass transfer rate in the presence of Soret coefficient when the partially heated active vertical left side wall has lower concentration while the opposite behaviour is observed when the partially heated active vertical left side wall has higher concentration. Alloui and Vasseur [10] studied analytically and numerically the double-diffusive and Soret-induced natural convection in a shallow rectangular cavity filled with a micropolar fluid. Nikbakhti and Rahimi [11] carried out a study of the double-diffusive natural convection in a rectangular cavity with partially thermally active side walls filled with air numerically and found that the average Nusselt number increases by increasing the buoyancy ratio, thus increasing the heat transfer rate in the cavity. Basu and Layek [12] gave a theoretical analysis to investigate the onset of double-diffusive convection at the marginal state in presence of cross-diffusive terms, viz. Soret and Dufour effects in a horizontal fluid layer and concluded that the Soret parameter destabilizes the oscillatory convection while the Dufour does the opposite. Sankar et al. [13] reported a numerical study of double-diffusive convection in a fluid-saturated vertical porous annulus subjected to discrete heat and mass fluxes from a portion of the inner wall. They discovered that the average Nusselt number increases with radius ratio; however, the average Sherwood number increases with radius ratio only up to when the radius ratio is equal to five, and for the radius ratio greater than five, the average Sherwood number does not increase significantly. Nik-Ghazali et al. [14] conducted a study on heat and mass transfer behaviour on porous medium embedded in a square annulus where the inner surface wall is considered to have a cool temperature while the outer surface is exposed to a hot temperature. They discovered that Soret effect tends to make contribution that is more significant to the concentration profile than Dufour effect. Cheng [15] studied the Soret and Dufour effects on the boundary layer flow due to free convection heat and mass transfer over a vertical cylinder in a porous medium saturated with Newtonian fluids with constant wall temperature and concentration. Their results showed that an increase in the Soret number leads to a decrease in the local Sherwood number and an increase in the local Nusselt number. The same author [16], in his study of free convection boundary layer flow over an arbitrarily inclined heated plate in a porous medium with Soret and Dufour effects, pointed out that an increase in the Dufour parameter tends to decrease the local heat transfer rate and an increase in the Soret parameter tends to decrease the local mass transfer rate. Ouriemi et al. [17] presented an analytical and numerical study of natural convection of a binary fluid confined in a tall enclosure, slightly inclined about the gravity field where they showed that, for a given Raleigh number, both "natural" flow, circulating clockwise and "anti-natural" flow, circulating anticlockwise are possible provided the angle of inclination is small enough. Lakshmi Narayana et al. [18] investigated the stability of Soret-driven thermosolutal convection in a shallow horizontal layer of a porous medium subjected to inclined thermal and solutal gradients of finite magnitude theoretically and observed