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Optimal variable shape parameters using genetic algorithm for radial basis function approximation

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Abstract Many radial basis function (RBF) methods contain free shape parameter or parameters that play an important role for the accuracy of the method. In most papers the authors end up choosing free shape parameter by trial and error or some other ad-hoc means. However, using variable shape parameters provides a clear potential for improved accuracy and stability of the RBF method. Already some progress has been reported to select usable variable shape parameters. In this paper, we propose applying the genetic algorithm to determine good variable shape parameters of radial basis functions for the solution of ordinary differential equations. Numerical results show that the proposed algorithm based on the genetic optimization is effective and provides reasonable shape parameters along with acceptable accuracy in linear and nonlinear case compared with other strategies to determine variable shape parameters.
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1. Introduction

Radial basis function (RBF) methods have been praised for their simplicity and ease of implementation in multivariate scattered data approximation [1], and they are becoming a viable choice as a method for the numerical solution of partial differential equations [2,3]. RBF-based methods offer numerous advantages, such as no need for a mesh or triangulation, simple implementation, dimension independence, and no staircasing or polygonization for boundaries. Moreover, depending on how the RBFs are chosen, high-order or spectral convergence can be achieved [4,5].

In this paper, we deal with finding reasonable variable shape parameters by taking a different approach. The purpose is to use a search algorithm using genetic optimization to determine good variable shape parameters c_i. In this study, the genetic optimization on the collocation method is tested to determine good c_i. Previously proposed strategies [6–9] and genetic algorithm strategy are used in collocation method in order to determine the optimal variable shape parameters in numerical solution of ordinary differential equations and the results are compared and analyzed. It should also be noted that Fornberg and Zuev [10], have used genetic algorithm in interpolation problem when investigating different shape parameters.

The layout of the article is as follows: In Section 2 we show that how we use the radial basis functions to approximate the

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solution. Then we briefly review the genetic optimization algorithm in next section. In Section 4, the promising genetic optimization algorithm for finding good variable shape parameters for solving ordinary differential equations by collocation method is introduced. The results of numerical experiments in linear and non-linear case are presented in Section 5.

2. Radial basis function approximation

Given a set of centers $X_1, \ldots, X_N$ in $\mathbb{R}^d$, the RBF approximation takes the following form:

$$S(X) = \sum_{i=1}^{N} \lambda_i \phi(r_i, c_i), \quad r_i = \|X - X_i\|_2.$$

We focus on RBFs $\phi(r_i, c_i)$ which are infinitely differentiable and contain free parameters c_i called the shape parameters. Some well-know radial basis function is presented in Table 1. The coefficients α are chosen by enforcing the interpolation condition

$$S(X_j) = f(X_j), \quad j = 1, \ldots, N,$$

on a set of nodes location which typically coincide with the centers. Enforcing the interpolation condition at N nodes location in an $N \times N$ linear system

$$A\Lambda = f,$$

to be solved for Gaussian expansion coefficients $\Lambda = [\lambda_1, \ldots, \lambda_N]^T$. The matrix A with the entries

$$a_{ji} = \phi(\|X_j - X_i\|_2), \quad j, i = 1, \ldots, N,$$

is called the interpolation matrix or the system matrix. It consists of functions serving as the basis of the approximation space. For distinct nodes location and coincide nodes location and centers, the system matrix for the positive definite RBFs (such as Gaussian radial basis function) is known to be nonsingular, if constant shape parameters used [11].

In all the numerical examples, we have used the Gaussian radial basis function ($\phi(r_i, c_i) = \exp(-c_i^2 r_i^2)$). The reason is that the Gaussian radial basis function interpolant has been shown to exhibit spectral convergence [12].

The accuracy and the stability for the infinitely smooth $\phi(x)$ depend on the number of data points and the value of the shape parameter c [13]. For a fixed c, as the number of data points increases, the RBF interpolation converges to the underlying (sufficiently smooth) function being interpolated at a spectral rate, i.e. $O(e^{-\frac{const.}{h}})$ where h is a measure of the "typical" distance between data points [14–16]. The value of *const* in the estimate is affected by the value of c. For a fixed number of data points, Madych [17] has shown that the accuracy of RBF interpolant can often be significantly improved by decreasing the value of c. However, decreasing c or increasing the number of data points has a severe effect on the stability of the linear system (2.4). For a fixed c, the condition number of the matrix in the linear system grows exponentially as the number of data points is increased [15,18]. For a fixed number of data points, as the shape parameter becomes small the condition number of the linear system grows [15,18].

Table 1 Some well-known functions that generate RBFs.

Name of function	Definition
Multiquadrics (MQ)	$\phi(r_i, c_i) = \sqrt{(c_i r_i)^2 + 1}$
Inverse multiquadrics (IMQ)	$\phi(r_i, c_i) = \left(\sqrt{(c_i r_i)^2 + 1}\right)^{-1}$
Gaussian (GA)	$\phi(r_i, c_i) = \exp\left(-c_i^2 r_i^2\right)$

Furthermore, there is a good approximation property for Gaussian RBF as follows:

Theorem 1. *For any $\alpha > 0$ and for any compact set Q in $\mathbb{R}^d$, the set of Gaussian radial functions*

$$\{x \longrightarrow e^{-\alpha\|x-y\|^2} : y \in Q\},$$

is fundamental in $C(Q)$.

Proof [19]. Recall that a set V in a normed linear space E is said to be fundamental if the closure of the span of V is E. In other words, the set of all linear combinations of elements of V is dense in E.

3. Genetic algorithm

The genetic algorithm (GA) is an optimization technique that is categorized as a global search heuristic. Genetic algorithms are based on an abstraction of the natural evolutionary behavior which was originally proposed in [20]. They are a robust and flexible approach that can be applied to a wide range of optimization problems (see, for example, [21–23]). Main aim of GA is to achieve better results by removing bad results during production of population from current generation to next generation and using only good results to achieve the better results. The fitness function that defines what is the "better" has to be prepared very carefully depending on the problem. Possible solutions in GA are presented by chromosomes and generally the first solutions are produced randomly. Chromosomes (individuals) together generate a set of solution populations.

The fitness function formed properly to the problem, presents a solution quality of the individuals. To produce new and good solutions, GA uses operators such as selection, crossover and mutation. To produce new generation crossover and mutation operators are applied on the two individuals that are selected from population by selection mechanism [24]. The genetic algorithm uses the individuals in the current generation to create the children that make up the next generation. Besides elite children, which correspond to the individuals in the current generation with the best fitness values, the algorithm creates crossover children by selecting vector entries, or genes, from a pair of individuals in the current generation and combines them to form a child and mutation children by applying random changes to a single individual in the current generation to create a child. Both processes are essential to the genetic algorithm. Crossover enables the algorithm to extract the best genes from different individuals and recombine them into potentially superior children. Mutation adds to the diversity of a population and thereby increases the likelihood that the algorithm will generate individuals with better fitness values. Crossover fraction specifies the fraction of the next generation that crossover produces. Mutation produces the remaining individuals in the next generation.

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