Journal of Magnetism and Magnetic Materials 372 (2014) 97–106

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Effects of heat transfer on peristaltic motion of Oldroyd fluid in the presence of inclined magnetic field

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ARTICLE INFO

Article history:
Received 19 April 2014
Received in revised form
18 July 2014
Available online 30 July 2014

Keywords:
Oldroyd fluid
Magnetic field
Heat transfer
Asymmetric channel
Nonlinear equations
Perturbation method
Analytical solutions

ABSTRACT

In this study the peristaltic motion of Oldroyd fluid in an asymmetric channel is investigated. Mathematical analysis has been carried out in the presence of an inclined magnetic field. Heat transfer is also taken into account. The physical problem is first modeled and then the analytical solutions of coupled equations are developed by regular perturbation method. Assumptions of long wavelength approximation are used. Effects of inclined magnetic field on the axial velocity and temperature are presented. Physical features of pertinent parameters such as wave number δ, Reynolds number Re, Weissenberg number *Wi*, Prandtl number Pr and Hartmann number *M* are also discussed graphically at the end of the paper.

1. Introduction

The peristaltic transport of fluid in a planar channel/tube has been examined extensively in view of its occurrence in industrial and physiological process. This type of flows are quite common in biological organs: for example, in urine transport from kidney to bladder, passage of food through esophagus, movement of chyme in the gastrointestinal tract, in ductus efferentes of male reproductive tract, in spermatozoa transport and several others. Industrial applications of such process include blood pumps in heat lung machine and sanitary and corrosive fluids transport. Latham [1] and Shapiro et al. [2] initially investigated the peristaltic flow of viscous fluid in a planar channel/tube. Afterwards a number of numerical, analytical and experimental studies have been conducted for the peristalsis under different conditions with reference to physiological and mechanical situations. Few recent studies in this direction include the works presented by Mekheimer [3], Tripathi [4], Elshehawey et al. [5], Mekheimer and Abdelmaboud [6], Haroun [7], Tripathi et al. [8], Ebaid [9], Kothandapani and Srinivas [10]. In continuation, Haroun [11] extend the discussion of peristaltic flow from the planer to inclined channel. He studied the peristaltic motion of fourth grade fluid in an inclined channel. Peristaltic flow in an inclined tube was addressed by Vajravelu et al. [12]. Peristaltic transport of viscous fluid in an inclined asymmetric channel is discussed by Srinivas and Pushparaj [13]. Ellahi et al. [14], Khan et al. [15] and Kothandapani and Srinivas [16] analyzed the peristaltic flow of viscous fluid in an inclined symmetric channel through a porous medium.

Moreover, magnetic field has gained great importance in industry and bio engineering, such as power generators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry and the purification of molten metals from nonmetallic inclusions and fluid-droplet sprays, etc. Furthermore blood flow with shear rate below 100 S^{-1} represents a mathematical model of peristaltic magnetohydrodynamics (MHD) flows in the coronary arteries.

Recently, the study of magnetic field in the presence of heat transfer has also received special status by the researchers. The MHD effects are especially useful in problems of the movement of conductive physiological fluids, the blood pump machines and in operation of peristaltic MHD compressor. Yldrm and Sezer [17] studied the effects of partial slip on the peristaltic flow of a MHD Newtonian fluid in an asymmetric channel. In order to see the wide range applications of magnetic, readers are referred to the studies [18–25]. Furthermore, the heat transfer analysis has vital role in processes of oxygenation and hemodialysis. Hence the interaction of heat transfer in peristaltic transport with different

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http://dx.doi.org/10.1016/j.jmmm.2014.07.051
0304-8853/

fluids is reported by Nadeem and Akbar [26]. Akbar et al. [27,28], Kothandapani and Srinivas [29], Mekheimer and Abdelmaboud [30], etc.

Additionally, the study of non-Newtonian fluids [31–36] is much more complicated and difficult because of the nonlinear relationship between the stress and the rate of strain occur in real world phenomena. It is very easy to solve a linear problem but finding a solution of nonlinear problem is still very challenging task. In particular, getting an analytic solution of a nonlinear problem is often more difficult as compared to getting a numerical solution, despite the availability of high performance supercomputers. However, results obtained by numerical methods give discontinuous points of a curve when plotted. Besides that, obtaining the complete necessary understanding of a nonlinear problem is very much difficult. If a nonlinear problem has multiple solutions or contains singularity then this also adds difficulties to the numerical. Though numerical and analytic methods for solving nonlinear problems have their own limitations, at the same time they have their advantages too. Therefore, we cannot neglect either of the two approaches but usually it is pleasing to solve a nonlinear problem analytically.

Motivated by these facts, the present work has been undertaken in order to analyze the peristaltic motion of Oldroyd fluid in an inclined asymmetric channel by regular perturbation method. Heat transfer and magnetic field are also taken into account. Contribution of the involved pertinent parameters for various parameters is discussed graphically. The results presented in this paper will be available for experimental verification to those who are not only interested to open new windows in this regime but also give confidence to them for the well-posedness of non-linear boundary value problems.

2. Mathematical formulation of the problem

We examine the peristaltic motion of an Oldroyd fluid in a two-dimensional inclined channel. The temperatures of the upper and lower walls of channel are T_0 and T_1 respectively. A magnetic field of constant strength B_0 is applied. Inclined status of channel and magnetic field is considered with the angles α and β respectively. The peristaltic motion in channel is induced because of sinusoidal waves propagating with wave speed c. Such waves are defined by the following expressions:

$$h_1(\overline{X},\overline{t}) = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right], \text{ upper wall} \tag{1}$$

$$\overline{h}_2(\overline{X},\overline{t}) = -d_2 - a_2 \cos\left[\frac{2\pi}{\lambda}(\overline{X} - c\overline{t}) + \phi\right], \text{ lower wall} \tag{2}$$

In the above expressions $a_i (i=1,2)$ denote the wave amplitudes, λ denote the wavelength, $d_1 + d_2$ is the channel width, ϕ $(0 \le \phi \le \pi)$ is the phase difference when $\phi = 0$ corresponds to symmetric channel with waves out of phase and for $\phi = \pi$, the waves are in phase. Also a_1, a_2, d_1, d_2 and ϕ obey the following relation:

$$a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi \le (d_1 + d_2)^2. \tag{3}$$

The constitutive equation for Oldroyd fluids is

$$\tau_{ij} + \Gamma\left[\begin{array}{c} \frac{\partial \tau_{ij}}{\partial t}\left(\sqrt{g^{kk} g_{ii} g_{jj}}\right)\nu_k \frac{\partial}{\partial x^k}\left(\sqrt{g^{ii} g^{jj}}\tau_{ij}\right) \\ -\sqrt{g^{kk} g_{jj}}\tau_{kj}\frac{\partial}{\partial x_k}\sqrt{g^{ii}}\nu_i - \sqrt{g^{kk} g_{ii}}\tau_{ik}\frac{\partial}{\partial x_k}\left(\sqrt{g^{jj}}\nu_j\right)\end{array}\right] = -\mu\dot{\gamma}_{ij}, \tag{4}$$

where g_{ii} and g^{ij} $(i,j = 1,2)$ are respectively the diagonal components of covariant and contravariant metric tensor [37].

The flow is unsteady in the laboratory frame $(\overline{X},\overline{Y})$. We move to the wave frame $(\overline{x},\overline{y})$ through the following transformations:

$$\overline{x} = \overline{X} - c\overline{t},\ \overline{y} = \overline{Y},\ \overline{u} = \overline{U} - c,\ \overline{v} = \overline{V}, \tag{5}$$

in which $(\overline{U},\overline{V})$ and $(\overline{u},\overline{v})$ are the velocity components in the corresponding coordinate system. The equations governing the flow in the wave frame of reference are

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{6}$$

$$\begin{aligned}\rho\left[\overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}}\right] &= -\frac{\partial \overline{p}}{\partial \overline{x}} - \frac{\partial \overline{\tau}_{\overline{xx}}}{\partial \overline{x}} - \frac{\partial \overline{\tau}_{\overline{yx}}}{\partial \overline{y}} + \rho g \sin\alpha \\ &\quad -\sigma B_0^2 \cos\beta((\overline{u}+c)\cos\beta - \overline{v}\sin\beta),\end{aligned} \tag{7}$$

$$\begin{aligned}\rho\left[\overline{u}\frac{\partial \overline{v}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{v}}{\partial \overline{y}}\right] &= -\frac{\partial \overline{p}}{\partial \overline{y}} - \frac{\partial \overline{\tau}_{\overline{xy}}}{\partial \overline{x}} - \frac{\partial \overline{\tau}_{\overline{yy}}}{\partial \overline{y}} - \rho g \cos\alpha \\ &\quad +\sigma B_0^2 \sin\beta((\overline{u}+c)\cos\beta - \overline{v}\sin\beta).\end{aligned} \tag{8}$$

The constitutive equations for Oldroyd fluids are

$$\overline{\tau}_{\overline{xx}} + \Gamma\left[\overline{u}\frac{\partial \overline{\tau}_{\overline{xx}}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{\tau}_{\overline{xx}}}{\partial \overline{y}} - 2\overline{\tau}_{\overline{xx}}\frac{\partial \overline{u}}{\partial \overline{x}} - 2\overline{\tau}_{\overline{xy}}\frac{\partial \overline{u}}{\partial \overline{y}}\right] = -\mu\dot{\overline{\gamma}}_{\overline{xx}}, \tag{9}$$

$$\overline{\tau}_{\overline{xy}} + \Gamma\left[\overline{u}\frac{\partial \overline{\tau}_{\overline{xy}}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{\tau}_{\overline{xy}}}{\partial \overline{y}} - 2\overline{\tau}_{\overline{yy}}\frac{\partial \overline{u}}{\partial \overline{x}} - 2\overline{\tau}_{\overline{xx}}\frac{\partial \overline{v}}{\partial \overline{x}}\right] = -\mu\dot{\overline{\gamma}}_{\overline{xy}}, \tag{10}$$

$$\overline{\tau}_{\overline{yy}} + \Gamma\left[\overline{u}\frac{\partial \overline{\tau}_{\overline{yy}}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{\tau}_{\overline{yy}}}{\partial \overline{y}} - 2\overline{\tau}_{\overline{xy}}\frac{\partial \overline{v}}{\partial \overline{x}} - 2\overline{\tau}_{\overline{yy}}\frac{\partial \overline{v}}{\partial \overline{y}}\right] = -\mu\dot{\overline{\gamma}}_{\overline{yy}}. \tag{11}$$

The energy equation is

$$\rho C_v\left[\overline{u}\frac{\partial \overline{T}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{T}}{\partial \overline{y}}\right] = k\nabla^2\overline{T} + \overline{\tau}_{\overline{xx}}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{\tau}_{\overline{yy}}\frac{\partial \overline{v}}{\partial \overline{y}} + \overline{\tau}_{\overline{xy}}\left[\frac{\partial \overline{u}}{\partial \overline{y}} + \frac{\partial \overline{v}}{\partial \overline{x}}\right], \tag{12}$$

where $\overline{p}$ is the pressure $\overline{\tau}_{\overline{xx}}$, $\overline{\tau}_{\overline{xy}}$, $\overline{\tau}_{\overline{yy}}$ are the components of extra stress tensor, Γ is the relaxation time, μ is the viscosity of the fluid, σ is the electrical conductivity, C_v is the specific heat at the constant volume, υ is the kinematic viscosity, k is the thermal conductivity of the fluid, $\overline{T}$ is the temperature and $\dot{\overline{\gamma}}_{\overline{xx}}$, $\dot{\overline{\gamma}}_{\overline{xy}}$, $\dot{\overline{\gamma}}_{\overline{yy}}$ are the components of strain rate tensor which are given by

$$\dot{\overline{\gamma}}_{\overline{xx}} = 2\frac{\partial \overline{u}}{\partial \overline{x}},\quad \dot{\overline{\gamma}}_{\overline{yy}} = 2\frac{\partial \overline{v}}{\partial \overline{y}},\quad \dot{\overline{\gamma}}_{\overline{xy}} = \frac{\partial \overline{u}}{\partial \overline{y}} + \frac{\partial \overline{v}}{\partial \overline{x}}. \tag{13}$$

We now introduce the following non-dimensional variables and parameters:

$$\left.\begin{array}{l} x = \frac{\overline{x}}{\lambda},\ y = \frac{\overline{y}}{d_1},\ u = \frac{\overline{u}}{c},\ v = \frac{\lambda\overline{v}}{d_1 c},\ h_1 = \frac{\overline{h}_1}{d_1},\ \tau_{ij} = \frac{d_1}{\mu c}\overline{\tau}_{ij},\ \delta = \frac{d_1}{\lambda} \\ p = \frac{d_1^2}{c\lambda\mu}\overline{p},\ h_2 = \frac{\overline{h}_2}{d_1},\ \mathrm{Re} = \frac{\rho c d_1}{\mu},\ Wi = \frac{\Gamma c}{d_1},\ M^2 = \frac{\sigma d_1^2 B_0^2}{\mu} \\ \mathrm{Pr} = \frac{C_v\mu}{k},\ E = \frac{c^2}{C_v(T_1 - T_0)},\ \theta = \frac{\overline{T}_1 - \overline{T}_0}{(T_1 - T_0)},\ \dot{\gamma}_{ij} = \frac{d_1}{c}\dot{\overline{\gamma}}_{ij}, \end{array}\right\}, \tag{14}$$

the non-dimensional Eqs. (6)–(12) in wave frame can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{15}$$

$$\left.\begin{array}{l}\mathrm{Re}\delta\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} - \delta\frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} + \frac{\mathrm{Re}}{Fr}\sin\alpha \\ \qquad -M^2\cos\beta((u+1)\cos\beta - \delta v\sin\beta)\end{array}\right\}, \tag{16}$$

$$\left.\begin{array}{l}\mathrm{Re}\,\delta^3\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} - \delta^2\frac{\partial \tau_{xy}}{\partial x} - \delta\frac{\partial \tau_{yy}}{\partial y} - \delta\frac{\mathrm{Re}}{Fr}\cos\alpha \\ \qquad +M^2\delta\sin\beta((u+1)\cos\beta - \delta v\sin\beta)\end{array}\right\}, \tag{17}$$

$$\left.\begin{array}{l}\tau_{xx} + Wi\left[\delta\left(u\frac{\partial \tau_{xx}}{\partial x} + v\frac{\partial \tau_{xx}}{\partial y} - 2\tau_{xx}\frac{\partial u}{\partial x}\right) - 2\tau_{xy}\frac{\partial u}{\partial y}\right] \\ = -2\delta\frac{\partial u}{\partial x}\end{array}\right\}, \tag{18}$$

$$\left.\begin{array}{l}\tau_{xy} + Wi\left[\delta\left(u\frac{\partial \tau_{xy}}{\partial x} + v\frac{\partial \tau_{xy}}{\partial y} - \delta\tau_{xx}\frac{\partial v}{\partial x}\right) - \tau_{yy}\frac{\partial u}{\partial y}\right] \\ = -\frac{\partial u}{\partial y} - \delta^2\frac{\partial v}{\partial x}\end{array}\right\}, \tag{19}$$