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Effect of vertical heterogeneity on the onset of ferroconvection in a Brinkman porous medium

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Abstract The onset of ferroconvection in a Brinkman porous medium is studied for different forms of vertical heterogeneity permeability function $\Gamma(z)$. The eigenvalue problem is solved numerically using the Galerkin method for isothermal/insulated rigid-ferromagnetic boundary conditions. It is observed that the onset of ferromagnetic convection can be either hastened or delayed depending on the type of heterogeneity of the porous medium as well as thermal boundary conditions. The effect of increasing magnetic number is to hasten the onset of ferroconvection for all choices of $\Gamma(z)$. Although the measure of nonlinearity of magnetization is to hasten the onset of ferroconvection in the case of isothermal boundaries, it shows no influence on the onset criterion when the boundaries are insulated. Further, the deviation in critical Darcy–Rayleigh number between different forms of $\Gamma(z)$ is found to be not so significant with an increase in the Darcy number.

1. Introduction

Ferrofluids (magnetic fluids) are commercially manufactured colloidal liquids usually formed by suspending mono-domain nanoparticles (their diameter is typically 10 nm) of magnetite in non-conducting liquids such as heptanes, kerosene, water, etc., and they are also called magnetic nanofluids. These fluids are magnetized in the presence of an external magnetic field, and due to their both liquid and magnetic properties, they have emerged as reliable materials capable of solving complex engineering problems. An authoritative introduction to this fascinating subject along with their applications is provided [1–3]. The magnetization of ferrofluids depends on the magnetic field, the temperature and the density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. This leads to convection in ferrofluids in the presence of magnetic field gradient, known as ferroconvection, which is similar to buoyancy driven convection. The theory of ferroconvection instability in a ferrofluid layer began

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Nomenclature

A	ratio of heat capacities	$\vec{q} = (u, v, w)$	velocity vector
a	horizontal wave number,	p	pressure
$D = d/dz$	differential operator	ρ_f	fluid density
$K(z)$	variable permeability of the porous medium	μ_f	fluid viscosity
$\vec{M}$	magnetization of the ferrofluid	$\tilde{\mu}_f$	effective viscosity
$\vec{H}$	magnetic field	μ_0	magnetic permeability
T	temperature	ρ_0	reference density
$Da = \tilde{\mu}_f K_0/\mu_f d^2$	Modified Darcy number	ε	porosity of the porous medium
$M_1 = \mu_0 K_p^2 \beta/(1+\chi)\alpha_t \rho_0 g$	Magnetic number	κ	thermal diffusivity
$M_3 = (1 + M_0/H_0)/(1 + \chi)$	Measure of nonlinearity of magnetization	α_t	thermal expansion coefficient
$R_D = \alpha_t g \beta K_0 d^2/\nu\kappa$	Darcy–Rayleigh number	$\beta = T_L - T_U/d$	temperature gradient
$\Gamma(z) = K_0/K(z)$	Permeability heterogeneity function	∇^2	Laplacian operator
K_0	Mean value of $K(z)$	*Subscripts*	
$\chi = (\partial M/\partial H)_{H_0, T_a}$	magnetic susceptibility,	f	fluid
$K_p = -(\partial M/\partial T_f)_{H_0, T_a}$	pyromagnetic co-efficient	b	basic state
$T_a = (T_L + T_U)/2$	average temperature of the fluid		

with Finlayson [4] and extensively continued over the years [5–10].

Thermal convection of ferrofluids saturating a porous medium has also attracted considerable attention in the literature owing to its importance in controlled emplacement of liquids or treatment of chemicals, and emplacement of geophysically imageable liquids into particular zones for subsequent imaging, etc. The stability of the magnetic fluid penetration through a porous medium in a high uniform magnetic field oblique to the interface is studied [11]. Thermal convection of ferrofluid saturating a porous medium in the presence of a vertical magnetic field is studied by several authors [12–18]. Recently, Nanjundappa et al. [19] investigated the onset of buoyancy and surface tension forces in a magnetized ferrofluid saturated horizontal Brinkman porous layer with temperature dependent viscosity.

The effect of heterogeneity in either permeability or thermal conductivity or both on thermal convective instability in a layer of porous medium is of importance since there can occur dramatic effects in the case of heterogeneity (Braester and Vadasz [20], Nield and Bejan [21] and references therein). The porous domain being heterogeneous is common in many engineering applications due to series of horizontal layers in each of which the permeability is uniform. Besides, flow of ferrofluids through porous media was motivated by the potential use of ferrofluids to stabilize fingering in oil recovery processes. In such situations the presence of heterogeneities is common and it may affect the flow of ferrofluids through porous media. Under the circumstances, investigating the influence of heterogeneity of permeability on ferromagnetic convection in a layer of porous medium is of practical interest and also warranted. However, it appears that majority of the studies on ferromagnetic convection in a porous medium deal with the case of homogeneous porous medium, and the case of heterogeneity has been largely neglected. In view of this, Shivakumara et al. [22] studied the onset of ferromagnetic convection in a heterogeneous Darcy porous medium using a local thermal non-equilibrium model.

Nonetheless, there is ample scope for further investigations on ferromagnetic convection in a heterogeneous porous medium. The intent of the present study is to emphasize various forms of vertical heterogeneity (property variation in the vertical direction) in the permeability of the porous medium on the onset of ferromagnetic convection in a magnetized ferrofluid-saturated horizontal layer of Brinkman porous medium. The resulting eigenvalue problem is solved numerically using the Galerkin method for isothermal/insulated rigid-ferromagnetic boundaries.

2. Mathematical formulation

We consider an incompressible magnetized ferrofluid-saturated horizontal layer of Brinkman heterogeneous porous medium of characteristic thickness d in the presence of a uniform applied magnetic field in the vertical direction. The lower surface is held at constant temperature T_L, while the upper surface is at $T_U\ (< T_L)$. A Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the porous layer and the z-axis directed vertically upward in the presence of gravitational field $\vec{g}$. The Boussinesq approximation on the density is made. The governing basic equations for thermal convection in a ferrofluid-saturated heterogeneous porous medium consist of the balances of mass, linear momentum, energy and they are respectively given by the following equation: [1,21]

$$\nabla \cdot \vec{q} = 0. \tag{1}$$

$$\rho_0 \left[\frac{1}{\varepsilon} \frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2} (\vec{q}.\nabla)\vec{q} \right] = -\nabla \mathrm{p} + \rho_0 \{1 - \alpha_t (T - T_a)\} \vec{g} - \frac{\mu_f}{K(z)} \vec{q} + \tilde{\mu}_f \nabla^2 \vec{q} + \mu_0 (\vec{M} \cdot \nabla) \vec{H} \tag{2}$$

$$A \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa \nabla^2 T \tag{3}$$